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13. ABSTRACT *(Maximum 200 Words)*

This Phase I project developed a trauma care classification method based on variables that can be easily ascertained in the field environment. The major achievements of the Phase I study include: (1) Establishment of a Gaussian Potential Function Network (GPFN) architecture that allows the discrimination between various classes representing the degree of severity of the trauma classification problem. These classes constitute the basis for field triage. The GPFN is configured as an aggregate of Guassian Potential Function Units (GPFUs); (2) Demonstration of the convergence properties of the training algorithm for the GPFN which adjusts the amplitudes, the means and the covariance matrices of the GPFUs to effect characterization of a given class as an integer value declaration; (3) Utilization of the fuzzy c-means clustering algorithm to partition the data into compact sets over which the GPFUs can be assigned. A cluster membership validity measure is also used to provide the fuzzy c-means algorithm with an estimate of the number of clusters present in the data; (4) A direct encoding classification method is also presented that allows the direct encoding of the prevalence of a given feature vector among the various classes.

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Chung-Fang Lu 10/4/96

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Chapter 1

Introduction

This final report documents the overall efforts and accomplishments of research and development of the Phase I project **Trauma Care Classification** undertaken by the American GNC (AGNC) Corporation for the U.S. Army Medical Research and Materiel Command, Fort Detrick, Frederick, MD and technically monitored by the U.S. Army Institute of Surgical Research, Mechanical Trauma Research Branch, San Antonio, Texas.

The objective of this Phase I project was to develop a processing architecture that accepts data from multiple inputs and provides likely trauma survival ratings. The processing architecture is based on a neural network configuration expanded to encode in a direct and unambiguous manner statistical information. This creates a hybrid architecture that permits the best attributes of both domains to be utilized for the classification of data related to trauma survival predictive variables. The approach is based on the realization that no single technique is capable of solving by itself the more difficult aspects of the highly complex trauma survival classification problem. Thus, there exists a strong need to coherently assemble the best elements of different techniques so as to reinforce the positive contributions by each and to neutralize, through complementation, their deficiencies. The architecture chosen is a Gaussian Potential Function Network (GPFN) consisting of Gaussian Potential Function Units (GPFU) with some key parameters determined by the statistical properties of the input feature vectors. Other network structural parameters are

determined through a "training" process that aims to yield a network output compatible to the object class the input vector belongs to.

1.1 Trauma Outcome Prediction Variables and Scores

The development of effective trauma classification techniques have a significant impact on the delivery of immediate medical care since they allow attention to be appropriately focused on the most severe cases and provide a logical maximization of the availability of limited resources. For example, in the military arena, reliable trauma classification can differentiate cases in the battlefield that can be dispensed with through field treatment versus the severe ones that dictate transportation to field hospitals.

There is an evident need, thus, to characterize a trauma patient in terms that relate to the probability or chance for survival. For this to occur appropriate features must be measured. There is a body of knowledge and experience, accumulated through the years, that provides useful guidance. Methods are directed at assessment of vital signs (such as, pulse, blood pressure and level of consciousness) as key determinants of organ and tissue damage. Variables are sought that are linked to cardiovascular, respiratory and central nervous system functions. For these variables to be effective it is deemed that they must possess certain properties that enhance their "intuitive" acceptability, i.e., there must be a reasonable association with probability of survival making them credible to experienced medical practitioners.

Variables that have been investigated as correlating with trauma care classification purposes include pulse, skin color, bleeding, injury region, injury type, respiratory rate, respiratory expansion, systolic blood pressure, capillary refill, eye opening, best verbal response and best motor response (Bever [2], Teasdale [15], Champion [4], Jennett [9], Morris [13]).

Various trauma scores have been created through the years in an attempt to capture by means of field measurable variables the degree of trauma severity. Among the most prominent efforts in this area are Dr. H. Champion's Trauma Score (TS), the Abbreviated Injury

Scale (A.I.S.) published in 1971 as a single comprehensive system for rating tissue damage sustained in motor-vehicle accidents, the Injury Severity Score (ISS) developed in 1974 to evaluate motor-vehicle victims with multiple injuries, the CRAMS scale and others. These scores attempt to categorize the degree of severity of trauma patients and some (such as the TS and CRAMS) are specifically designed for field triage of trauma victims to trauma centers.

Physical examination for the purpose of increasing the diagnostic precision has obvious limitations in environments such as the battlefield. Thus, the variables sought for trauma outcome prediction in a coarse field environment must exclude those that can be ascertained through the more sophisticated tools and practices available in a specialized hospital setting.

1.2 Statistics and Neural Networks

Statistical considerations have been the most prominent in the long history of data classification. Their mathematical formalism is well developed and numerous application studies complement the theoretical pronouncements (Lin [11], Lin[12], Fukunaga[6]). In all scientific disciplines there is a steady requirement to automate the data classification process. Although numerous classification techniques have been formulated over the years no method has demonstrated clear superiority. The common concensus is that each problem invariably presents its own intricate details and thus particularizes its classification approach. The volume of techniques and approaches is thus of benefit to the designer of a specific effort since he has now available a wealth of tools to tap for his problem. The data classification literature does distinguish gross stratification of techniques such as parametric and nonparametric, supervised and nonsupervised and deterministic versus statistical. However, cross-fertilization of ideas among the various classification categories constantly takes place and sometimes it is very difficult to clearly demarcate them.

The statistical classification problem has a clear separation into two phases. The first isolates from raw data characterizations appropriately computed subsets that are deemed

pertinent to the problem at hand. This is commonly referred to as the feature extraction phase. The second phase creates feature partitioning functions so that feature data from different classes can be clearly separated from each other. This is commonly referred to as the classification phase. Experience indicates that the feature definition phase is most crucial to the pattern classification efforts,, although it should be noted that a poorly designed classifier can easily ruin the potential benefits of a well designed feature set.

The data classification discipline has been heavily influenced by statistical considerations and justifiably so since its roots lie in statistics. However, the ultimate successful embedding of a classification problem in a statistical framework requires assumptions (such as normality) which may not always be valid for the data at hand. Although the methodologies may be optimal under assumed conditions, the data may simply not fulfill the implied hypotheses. Several attempts have been made to abandon restrictive probability distributional assumptions but no method has emerged as a clear alternative. Although the statistical classifiers have shown sufficient success to remain at the top of the useful classification tools, new and effective aiding techniques are always in demand. Such techniques emerged with the advent of neural networks.

One important attribute of neural networks is that neural networks are learning systems. Furthermore, artificial neural network structures can be easily implemented in hardware that has a large scale, robust and parallel computational power. Massive parallelism in computational networks is extremely attractive in principle. But in practice there are many important issues to be addressed before a successful implementation can be achieved for a given problem. In the following, we present some prominent issues which are crucial to the success of practical neural network implementations.

The representation ability of multilayer feedforward networks has been investigated over the past few years. There are many papers on this subject. These include Cybenko [5], Hornik [7], Hornik [8], Poggio [14] and many others. Their results show that any continuous functions can be approximated arbitrarily well by a layered network with one

hidden layer, where the hidden nodes represent either sigmoidal functions Cybenko [5], Hornik [7], Hornik [8], or radial basis functions Poggio [14]. This conclusion holds under the condition that there are a sufficiently large number of hidden units. The concept of using neural networks as modeling tools has been tested over many practical cases in different areas. In basically all applications of artificial neural networks, the networks are built almost entirely by trial and error. Some guidance is gained from prior experience as to the number and arrangement of neurons and the particular parameters used in training, such as learning rate and momentum.

Many other researchers consider the comprehensive design of neural networks in a variety of ways. Some use an expert system, some use genetic algorithms to evolve a superior network for a given application, some design the neural network with hierarchical structures that are related to the known structure of the application and some combine neural networks with fuzzy sets to advantage.

Neural networks are much better interpolators than extrapolators. Although we still do not have a comprehensive guideline for the design of artificial neural networks, it has been demonstrated by numerous successful cases that by trial and error, one can always come up with a neural network model which associates certain inputs to certain outputs even under noise corruption.

The reason for much of the appeal of neural networks is their ability to generalize to a new situation. This is a very useful property in applications since all the measurements will not be the same for different occasions even under very similar operating conditions.

After being trained on a number of examples of a relationship, neural networks can often define a complete relationship that interpolates and extrapolates from the examples in a sensible way. But what is meant by sensible generalization is often not clear. In many problems there are almost infinitely many possible generalizations. How does a neural network - or a human for that matter - choose the "right" one? We should know what we are expecting a network to do when we look for generalization.

Neural networks have shown excellent capabilities in encoding large amounts of information and provide even more beneficial attributes in their ability to accommodate new information through "learning" algorithms. However, most neural network architectures suffer an interpretation problem. In other words, their behavior is that of a black box that makes it almost impossible to decipher the cause and effect internal interactions that lead to the external manifestations. Basically, the network's behavioral traits are accepted as such with no easy linkage to their interconnection properties. This is to be expected given the vast number of weights and involved feedforward and feedback computational links.

In this project we employ a neural network configuration consisting of an assemblage of gaussian functions. The architecture chosen allows the exploitation of both statistical information and the "training" benefits of neural networks. It is also easy to geometrically visualize, in contrast to the other neural network architectures. The approach was motivated by the realization that no single technique is capable of solving by itself the more difficult aspects of the highly complex trauma survival classification problem. Thus, there exists a strong need to coherently assemble the best elements of different techniques so as to reinforce the positive contributions by each and to neutralize, through complementation, their deficiencies.

1.3 Results of the Phase I Work

The Phase I project aimed at the development of an integrated trauma care classification system within the context of measurable variables that can be easily ascertained in the field environment. The research involved clustering and neural network based methodologies that can accommodate the practical aspects of the trauma care classification domain. The objectives were to: (1) establish a neural network based architecture that allowed through training to capture the detailed feature space based distribution of trauma care related feature vectors; (2) Improve the efficiency of the neural network based classification methodology through the use of the fuzzy c-means clustering algorithm; (3) Enhance

the effectiveness of the fuzzy c-means clustering algorithm through utilization of membership validity measures, and (4) Employ a direct data encoding approach utilizing gaussian functions to provide a direct and unambiguous statistical summary of the probabilistic prevalence of the observed feature vectors.

Specifically, the major achievements of the Phase I study include the following:

- Establishment of a Gaussian Potential Function Network (GPFN) architecture that allows the discrimination between various classes representing the degree of severity of the trauma classification problem. These classes constitute the basis for field triage. The GPFN is configured as an aggregate of Guassian Potential Function Units (GPFUs) that are positioned at the mean of the data distributions and along the feature axes at distances which are functions of the standard deviation per feature axis.

- Demonstration of the convergence properties of the training algorithm for the GPFN which adjusts the amplitudes, the means and the covariance matrices of the GPFUs to effect characterization of a given class as an integer value declaration.

- Utilization of the fuzzy c-means clustering algorithm to partition the data into compact sets over which the GPFUs can be assigned. Since the fuzzy c-means clustering algorithm requires the a priori specification of the expected number of clusters a membership validity measure is invoked whose minimum value is configured as a general indication of the most likely number of clusters present in the data set.

- A direct encoding classification method is presented that assigns a gaussian function to each data point of a given class. This method, encountered in probability density estimation studies, allows the direct encoding of the prevalence of a given feature vector among the various classes.

The report is organized as follows:

In Chapter 2, to alleviate the difficult interpretation problem of the established neural network architectures and to provide a more tractable mathematical foundation, the basic element for the classification configuration considered is the Gaussian Potential Function Unit (GPFU). The collection of several GPFUs constitutes a Gaussian Potential Function Network (GPFN). The GPFN synthesizes a potential field by allocating a set of Gaussian functions at selected points of an input feature space (Lee and Kil [10]). The "potential field", created by a GPFN, is to be utilized as a pattern discrimination tool based on a training phase which adjusts the magnitude, position and covariance characteristics of each GPFU so that a specific desired answer results for input vectors that are known to belong to a given object class.

Chapter 3 presents the classification approach within the context of specific examples illustrating the efficacy of the GPFN architecture. In a classification problem the first act is the definition of classes that we want to partition our problem into. The next step is to train the classifier to yield the correct mapping from feature vectors to output classes for a set of data for which this relation is known from past experience. Thus, this classification training phase adjusts the parameters of the classifier so that it properly performs in the desired manner (i.e., high correct classification rate). The classification technique represented by the GPFN assigns to a given class a distinct output value. Thus for a multiclass problem we would choose a different integer value (say, 1,2,3, etc.) for each class.

Chapter 4 examines the fuzzy c-means clustering algorithm as an aid to the GPFN classification algorithm. Clustering represents one of the broader and most sought after data analysis techniques. The vast appeal of clustering techniques has to do with the fact that realistic data structures are often the aggregate of a disjoint set of data groups, as so characterized by common consensus in visual observations, at least for low dimensionality feature vectors where such visual appraisals can be directly executed. Clustering can become a classification technique all by itself. However, for our purposes clustering is to act as a preprocessing method that allows identification of compact groups of data that Gaussian Potential Function Units can be defined for. Thus, clustering represents a band-

width compression technique for us. The clustering algorithm we chose is the fuzzy c-means algorithm developed by Dunn [17] and extended by Bezdek [3]. It is the most prominent fuzzy clustering algorithm with significant applications in the biomedical area [1].

Chapter 5 presents results from real data obtained from the National Study Center for Trauma and EMS at the University of Maryland. Trianalytics, Inc., which maintains the data base for the University of Maryland, provided us with 200 records corresponding to 100 penetrating (gunshot) wound records for male patients who survived and 100 penetrating (gunshot) wound records for male patients who did not survive. The patient population age was around 25-30 years. Also the patients had no preexisting conditions. The fuzzy c-means clustering and the GPFN classification approach were exercised with this data. In addition, a direct encoding classification technique is presented where a GPFU of unit variance was utilized for each entry from the surviving class of patients and for each of the patients of the nonsurviving class. The gaussians were added to create two surfaces in feature space that effectively summarized the probabilistic prevalence of a feature vector in feature space. This method, encountered in probability density estimation studies provides a direct and unambiguous classification methodology.

Chapter 6 draws conclusions from this study and also provides recommendations for future efforts.

Chapter 2

Gaussian Potential Functions

To alleviate the difficult interpretation problem of the established neural network architectures and to provide a more tractable mathematical foundation the basic element for the classification configuration considered in this project is the Gaussian Potential Function Unit (GPFU) which is a Gaussian function:

$$\psi(\overline{x}, \overline{\mu}, \Sigma) = e^{-\frac{1}{2}(\overline{x}-\overline{\mu})^T \Sigma^{-1} (\overline{x}-\overline{\mu})} \tag{2.1}$$

that assigns a functional value ψ to an arbitrary input vector $\overline{x}$. We have used the notation $\overline{\mu}$ and Σ from the statistical literature to denote the center value and the dispersion of the Gaussian function although no connection with statistical means and covariances may occasionally be in effect. Thus, although sometimes it might be conceptually slightly disagreeable, this notational convention is so prevalent that it deemed to us unnecessary to deviate from accepted practice. However, in the context of a specific discussion we shall be careful to clarify the exact meaning of the specific $\overline{\mu}$ and Σ under consideration.

The collection of several GPFUs constitutes a Gaussian Potential Function Network (GPFN). The GPFN synthesizes a potential field by allocating a set of Gaussian functions at selected points of an input feature space (Lee and Kil [10]). A summation function of GPFUs is then created.

$$\phi(\overline{x}) = \sum_{i=1}^{M} c(i)\psi(\overline{x}, \overline{\mu}_i, \Sigma_i) = \sum_{i=1}^{M} c(i) e^{-\frac{1}{2}(\overline{x}-\overline{\mu}_i)^T \Sigma_i{}^{-1}(\overline{x}-\overline{\mu}_i)} \tag{2.2}$$

The "potential field", created by a GPFN, is to be utilized as a pattern discrimination tool based on a training phase which adjusts the magnitude, position and covariance characteristics of each GPFU so that equation (2) yields a specific desired ϕ answer for input vectors that are known to belong to a given object class. The adjustment of the amplitude c(i), mean vector $\overline{\mu}_i$ and covariance matrix Σ_i of each GPFU is accomplished through the following training process.

An input feature vector $\overline{x}$, from a set of predictive variables, is presented to the GPFN and the output $\phi(\overline{x})$ noted. If this output is not equal to the desired output value, $\phi_{desired}(\overline{x})$, an error, E, ensues

$$\begin{aligned} E &= \frac{1}{2}(\ \phi_{desired}(\overline{x}) - \phi(\overline{x})\ )^2 \\ &= \frac{1}{2}(\ \phi_{desired}(\overline{x}) - \sum_{i=1}^{M} c(i) e^{-\frac{1}{2}(\overline{x}-\overline{\mu}_i)^T \Sigma_i{}^{-1}(\overline{x}-\overline{\mu}_i)}\ )^2 \end{aligned} \tag{2.3}$$

Now the goal of training consists in modifying the various parameters under the designer's control, i.e. the amplitude components c(i), the elements of the mean vectors $\overline{\mu}_i$ and the elements of the covariance matrices Σ_i of the GPFUs, in a direction that tends to minimize the error equation (2.3). The error is minimized through a gradient-descend process requiring that the various parameters be updated in proportion to the negative partial derivative of the error function (2.3) with respect to the parameter of interest. Thus, by evaluating the partial derivatives, the weights c(1),c(2),...,c(M), the elements of the mean vectors $\mu_1^1, \mu_2^1, ..., \mu_n^1, \mu_1^2, \mu_2^2, ..., \mu_n^2, ..., \mu_1^M, \mu_2^M, ..., \mu_n^M$ and the elements of the shape matrices K_i (the inverse of the covariance matrix Σ_i), $k_{11}^1, k_{12}^1, ..., k_{nn}^1, k_{11}^2, k_{12}^2, ..., k_{nn}^2, ..., k_{11}^M, k_{12}^M, ..., k_{nn}^M$ are modified according to the formulas

$$\begin{aligned} New\ c(i) &= Old\ c(i) + \eta(-\frac{\partial E}{\partial c(i)}) \\ &= Old\ c(i) + \eta(\phi_{desired}(\overline{x}) - \phi(\overline{x})) e^{-\frac{1}{2}(\overline{x}_i-\overline{\mu}_i)^T \Sigma_i^{-1}(\overline{x}_i-\overline{\mu}_i)} \end{aligned}$$

$$
\begin{aligned}
New\ \mu_j^i &= Old\ \mu_j^i + \eta(-\frac{\partial E}{\partial \mu_j^i}) \\
&= Old\ \mu_j^i \\
&\quad + \eta(\phi_{desired}(\overline{x}) - \phi(\overline{x}))(\ \sum_{j=1}^{n}(x_j - \mu_j^i)(k_{ij} + k_{ji}\ )\ ) \times \\
&\quad \times\ c(i)e^{-\frac{1}{2}(\overline{x}_i - \overline{\mu}_i)^T \Sigma_i^{-1}(\overline{x}_i - \overline{\mu}_i)} \\
New\ k_{jl}^i &= Old\ k_{jl}^i + \eta(-\frac{\partial E}{\partial k_{jl}^i}) \\
&= Old\ k_{jl}^i \\
&\quad + \eta(\phi_{desired}(\overline{x}) - \phi(\overline{x}))(x_j - \mu_j^i)(x_l - \mu_l^i\ ) \times \\
&\quad \times\ c(i)e^{-\frac{1}{2}(\overline{x}_i - \overline{\mu}_i)^T \Sigma_i^{-1}(\overline{x}_i - \overline{\mu}_i)} \qquad (2.4)
\end{aligned}
$$

where η is a positive constant called the learning rate.

Repeated iteration of the above described parametric update process over the set of input vectors structures a specific distribution of these GPFUs over the input space such that the error between the desired output and the actual output is minimized. If sufficient exemplars of the input object class have been utilized during this training phase then the GPFN can act as a pattern classifier by responding with the desired output for an input that originated from the same object class but was not part of the training set.

2.1 Gaussian Potential Function Network

2.1.1 Classification with a GPFN (Training Phase)

The first phase of a classification problem consists of the definition of a feature vector that has as elements the N variables that are deemed essential to characterize the problem at hand. For example, trauma characterization may involve variables such as pulse, skin color, bleeding, injury region, injury type, respiratory rate, respiratory expansion, systolic blood pressure, capillary refill, eye opening, best verbal response and best motor response. Often the feature variables are restricted in number, due to practical considerations, to a

subset of what would be ideally desired. In general then, the feature vector, $\overline{x}$, is defined as $\overline{x} = (x_1, x_2, ...x_N)$ where $x_1, x_2, ...x_N$ are the individual feature variables.

The second phase of the classification problem defines a set of classes that are deemed pertinent to the problem under investigation. For example, we could partition the prediction of the ultimate future state of the trauma victim into, say, three classes: Highly likely to survive (Class 1), likely to survive (Class 2) and unlikely to survive (Class 3). One may even be able to dispense with a discrete stratification of outcomes and utilize a continuous scale, such as, probability of survival, p, with the interpretation that in a large series of observations the observed feature vector is expected to manifest itself $p \times 100$ percent of the time (i.e. if $p = 0.1$, for example, then a feature vector associated with such a probabilistic manifestation implies that $0.1 \times 100 = 10$ percent of the trauma victims characterized with such a feature vector are expected to survive). Discretization of the classification problem into a number of distinct classes, instead of a continuous classification outcome, is often encountered in practice due to considerations that deem such a partitioning to be adequate for whatever other actions are further needed for the problem under consideration.

Having defined the features that constitute the feature vector and having established the number of classes that we want to partition our problem into, the next step is to train the classifier to yield the correct mapping from feature vectors to output classes for a set of data for which this relation is known from past experience. Thus, this classification training phase adjusts the parameters of the classifier so that it properly performs in the desired manner. The parameters to be adjusted depend on the design of the classifier. Following this training phase the classifier is to be used as a future predictive tool to yield the right answers for feature vectors for which the classification outcome is not known. This is the, so called, testing phase of the classifier. It represents its generalization capability. In an intuitive sense, one desires that the training phase consists of data sufficient to capture the statistical essence in both depth and breadth of the problem under consideration. As an analogy, it is not reasonable to expect that troops trained only for mountainous operations will adequately perform during amphibious ones.

It is assumed that training data for a given class are available. The classification technique represented by the GPFN configuration now involves two key considerations.

First it is noted that an input object to the classifier is characterized by a set of N features constituting its "signature". For the trauma classification problem it can be safely assumed that each feature assumes a set of discrete values. For example, the respiratory rate feature can be discretized into, say, five discrete numerical outcomes, 1 (1 to 9 breaths per minute), 2 (10 to 24 breaths per minute), 3 (25 to 35 breaths per minute), 4 (36 or more breaths per minute) and 5 (no breaths). In other scientific areas where continuous variables are encountered (say, a voltage measurement) it is again feasible to discretize the feature range through appropriate interval definitions at any desired level of refinement.

Let it be assumed that there are N total features and that the ith feature is represented by n_i discrete values. When the N features are considered in combination they define the feature vector $\overline{x} = (x_1, x_2, ...x_N)$. Since feature 1 can assume n_1 values, feature 2 n_2 values,..., and feature N n_N values, the total number of possible N-dimensional manifestations of the feature vector is L = $n_1 \times n_2 \times ... \times n_N$. For example, if we have three features and feature 1 is characterized by 10 values, feature 2 by 20 values and feature 3 by 30 values then the total number of possible feature vectors is L = 10×20×30 = 6000.

The feature vector is now input to a set of M unit amplitude GPFUs. The selection of a unit amplitude is arbitrary and corresponds to a desired classification answer of 1 for the specific survival class under consideration. For a multiclass problem we would choose several sets of M GPFUs, in parallel, with each parallel set designed to yield different integer values (say, 1,2,3, etc.) for each class.

Statistical considerations are now invoked to structure a total of M = 2N + 1 GPFUs by first centering one GPFU (i.e. setting its center value) at the nominal mean value of the feature vector as determined by the training data. In other words, the N-dimensional center vector of this first GPFU has as elements the mean values of the features.

Following the definition of the first GPFU, 2N additional GPFUs are next considered,

positioned symmetrically along each N-dimensional axis (with each axis associated with a feature) at a distance δ from the nominally centered first GPFU. The distance δ varies, in general, per axis and is equal to the standard deviation σ of each feature. It should be noted that it is straightforward, if needed by the data structure, to either refine or extend the expanse of the GPFUs by, for example, positioning them at intervals of $\sigma/2$ or 2σ, say.

The initial (prior to training) covariance matrices of all GPFUs are made equal to diagonal matrices with diagonal elements the variances σ^2 of the features. The use of diagonal matrices is for convenience. It is easy to incorporate off diagonal terms if there exist feature crosscorrelations.

The above discussion set the initial conditions of the GPFUs as regards the amplitudes c(i), mean vectors $\overline{\mu}_i$ and covariance matrices Σ_i. The training phase first considers all the sample objects and calculates the statistics necessary to establish the centering parameters and the dispersion matrices of the GPFUs. Then, feature vectors are iteratively provided as inputs to the GPFUs. Now, an error correction process takes place which alters the amplitudes c(i), mean vectors $\overline{\mu}_i$ and covariance matrices Σ_i of the GPFUs so as to achieve minimization of the discrepancy between desired classification output and actual output. The number of iterations through the training samples to achieve minimum error can not be theoretically predicted. It is experimentally determined.

2.1.2 Classification with a GPFN (Testing Phase)

Following the training phase a classifier is evaluated on the basis of a testing phase. The testing phase evaluates the correct classification rates for feature vectors that were not included in the training phase. This process characterizes the efficacy of the classifier's design and ascertains that the training phase captured the essential statistical basis of the problem at hand. Thus, in the testing phase, if the classification response to the testing feature vectors is close (to within a predetermined threshold) to the expected output value for the survival class under consideration (say, 1), then, the feature vector under test is

declared as a member of the class and a correct classification outcome is noted.

2.1.3 Example

Here we present an example that demonstrates the quantitative properties of the GPFN classification technique.

Four arbitrary features are considered. The nominal values of these features are taken to be 9, 5, 17 and 22, respectively. Noise which is uniform in the interval 0 to 3 is utilized to generate variations of these values for ten training feature vectors:

$$\begin{array}{ll}
\textit{train vector } 1 & (x_1, x_2, x_3, x_4) = (8.16,\ 5.09,\ 17.08,\ 21.48) \\
\textit{train vector } 2 & (x_1, x_2, x_3, x_4) = (7.64,\ 5.51,\ 15.77,\ 22.39) \\
\textit{train vector } 3 & (x_1, x_2, x_3, x_4) = (9.54,\ 3.52,\ 17.46,\ 22.77) \\
\textit{train vector } 4 & (x_1, x_2, x_3, x_4) = (9.53,\ 4.65,\ 16.74,\ 23.47) \\
\textit{train vector } 5 & (x_1, x_2, x_3, x_4) = (10.3,\ 3.70,\ 17.60,\ 21.59) \\
\textit{train vector } 6 & (x_1, x_2, x_3, x_4) = (8.65,\ 4.75,\ 18.23,\ 21.24) \\
\textit{train vector } 7 & (x_1, x_2, x_3, x_4) = (9.05,\ 5.56,\ 17.78,\ 23.44) \\
\textit{train vector } 8 & (x_1, x_2, x_3, x_4) = (9.99,\ 5.26,\ 16.28,\ 22.66) \\
\textit{train vector } 9 & (x_1, x_2, x_3, x_4) = (7.60,\ 6.29,\ 15.64,\ 22.76) \\
\textit{train vector } 10 & (x_1, x_2, x_3, x_4) = (7.66,\ 6.04,\ 17.70,\ 22.45)
\end{array}$$

These ten feature vectors are used to train the GPFN classifier. The mean and standard deviation for each feature for the ten training feature vectors are:

$$Feature\ 1 : Mean :\ 8.81\quad Standard\ Deviation :\ 0.97$$

$$Feature\ 2 : Mean :\ 5.04\quad Standard\ Deviation :\ 0.86$$

$$Feature\ 3 : Mean : 17.03 \quad Standard\ Deviation :\ 0.84$$

$$Feature\ 4 : Mean : 22.42 \quad Standard\ Deviation :\ 0.73$$

The GPFUs are selected by first centering a GPFU at the four-dimensional (since there are four features) vector

$$GPFU\ No.1\ center :\ \ (8.81,\ 5.04,\ 17.03,\ 22.42) \tag{2.5}$$

with elements the corresponding mean vectors of each feature.

Eight more GPFUs (two per axis in the four-dimensional feature space) are centered at a distance of one standard deviation (for the corresponding to the axis feature) from the first GPFU :

$$GPFU\ No.2\ center :\ \ (9.78,\ 5.04,\ 17.03,\ 22.42)$$

$$GPFU\ No.3\ center :\ \ (7.84,\ 5.04,\ 17.03,\ 22.42)$$

$$GPFU\ No.4\ center :\ \ (8.81,\ 5.90,\ 17.03,\ 22.42)$$

$$GPFU\ No.5\ center :\ \ (8.81,\ 4.17,\ 17.03,\ 22.42)$$

$$GPFU\ No.6\ center :\ \ (8.81,\ 5.04,\ 17.87,\ 22.42)$$

$$GPFU\ No.7\ center :\ \ (8.81,\ 5.04,\ 16.18,\ 22.42)$$

$$GPFU\ No.8\ center :\ \ (8.81,\ 5.04,\ 17.03,\ 23.16)$$

$$GPFU\ No.9\ center :\ \ (8.81,\ 5.04,\ 17.03,\ 21.69)$$

The dispersion of the GPFUs is set equal to a diagonal matrix with each diagonal element being equal to the variance of the corresponding feature.

The desired classification output is set to 1 and an iterative training phase follows to adjust the magnitudes c(i), mean vectors μ_i and covariance matrices Σ_i of each GPFU to achieve minimum error between the desired classification output (equal to 1) and the actual output. Without training the resulting average squared error from the ten vectors is 0.8402. After 500 iterations of the training algorithm the average squared error drops to 0.0248, an improvement by a factor of 33.

2.2 Learning Rate

As previously mentioned, the GPFN synthesizes a potential field by allocating a set of Gaussian functions at selected points of the input feature space. A summation function of gaussian functions is then created

$$\phi(\overline{x}) = \sum_{i=1}^{M} c(i)\psi(\overline{x}, \overline{\mu}_i, \Sigma_i) = \sum_{i=1}^{M} c(i) e^{-\frac{1}{2}(\overline{x}-\overline{\mu}_i)^T \Sigma_i{}^{-1}(\overline{x}-\overline{\mu}_i)} \tag{2.6}$$

to be utilized as a pattern discrimination tool based on a training phase which adjusts the magnitude, position and covariance characteristics of each GPFU so that the above expression yields a specific desired ϕ answer for input vectors that are known to belong to a given object class. The adjustment of the amplitude c(i), mean vector $\overline{\mu}_i$ and covariance matrix Σ_i of each GPFU is effected through equations 2.4 which involve the learning rate constant η. The learning rate constant is a key variable to the iteration process but unfortunately there is no theoretical method to establish its value for a given problem. We have determined experimentally a nominal range of feature values and a set of values for the constant η that show good results. The following description presents the details of our efforts.

Let it be assumed that there are N total features and that the mean of the ith feature is m_i. When the N features are considered in combination they define the feature vector $\overline{x} = (x_1, x_2, ...x_N)$. We first bias off the means m_i to a predetermined value M. This can be effected through the transformation $X_i = x_i +$ (M - m_i). The reason for selecting a constant value M for all features is to symmetrize the distribution with respect to the feature axes and thus enhancing the validity of a non directionaly dependent constant learning rate. We have selected the value 10 for M.

We next considered a nominal three dimensional feature vector of three elements: (10,10,10). We then generated 500 feature vectors by perturbing the nominal vector with a random value in the range -10 to 10. The 500 feature vectors are shown in Figure 2.1. The feature

vectors are plotted as waveforms for illustration purposes. The elements of the feature vectors are the ordinate values corresponding to the integer abscissa values 1,2 and 3. The mean and standard deviation for each feature for the 500 feature vectors are:

$$\begin{aligned}
&Feature\ 1 : Mean :\ 9.9392\quad Standard\ Deviation :\ 5.6367\\
&Feature\ 2 : Mean : 10.0160\quad Standard\ Deviation :\ 5.5954\\
&Feature\ 3 : Mean :\ 9.8471\quad Standard\ Deviation :\ 5.7422
\end{aligned} \tag{2.7}$$

Since there are three features we have a corresponding set of 7 GPFUs centered at the mean value of the distribution and at one sigma distances from the mean in each direction for each feature axis.

The GPFUs are selected by first centering a GPFU at the three-dimensional (since there are three features) vector

$$GPFU\ No.1\ center :\ (9.9392,\ 10.0160,\ 9.8471) \tag{2.8}$$

with elements the corresponding mean vectors of each feature.

Six more GPFUs (two per axis in the three-dimensional feature space) are centered at a distance of one standard deviation (along each feature axis) from the first GPFU :

$$\begin{aligned}
&GPFU\ No.2\ center :\ (15.5759, 10.0160,\ 9.8471)\\
&GPFU\ No.3\ center :\ (\ 4.3025, 10.0160,\ 9.8471)\\
&GPFU\ No.4\ center :\ (\ 9.9392, 15.6115,\ 9.8471)\\
&GPFU\ No.5\ center :\ (\ 9.9392,\ 4.4206,\ 9.8471)\\
&GPFU\ No.6\ center :\ (\ 9.9392, 10.0160, 15.5893)\\
&GPFU\ No.7\ center :\ (\ 9.9392, 10.0160,\ 4.1048)
\end{aligned} \tag{2.9}$$

The shape matrices (inverse of the covariance matrices) of the GPFUs are initially set equal to a diagonal matrix with each diagonal element being equal to the inverse of the variance of the corresponding feature:

$$\begin{bmatrix} 0.0315 & 0 & 0 \\ 0 & 0.0319 & 0 \\ 0 & 0 & 0.0303 \end{bmatrix} \tag{2.10}$$

The learning rate was adjusted experimentally to have the following values:

$$\begin{aligned} \eta_1 &= 0.00025 \\ \eta_2 &= 0.00025 \\ \eta_3 &= 0.0000025 \end{aligned} \tag{2.11}$$

We next trained the network for 300 iterations. The resulting error time history is shown in Figure 2.2. It is seen that significant improvement results from the learning phase. The initial error (i.e. the square of the difference between the desired GPFN value of 1 and the actual output) is 3.6675. After 300 iterations the error has dropped to a value of 0.0477. The learning phase has resulted into the mean vectors of the GPFUs shifting to new positions as follows:

$$\begin{aligned} New\ GPFU\ No.1\ center: &\ (\ 9.8884,\ 9.9543,\ 9.8158) \\ New\ GPFU\ No.2\ center: &\ (16.2617, 10.6919, 10.4892) \\ New\ GPFU\ No.3\ center: &\ (\ 3.6272,\ 9.3740,\ 9.2284) \\ New\ GPFU\ No.4\ center: &\ (\ 9.9193, 16.2078, 10.4552) \\ New\ GPFU\ No.5\ center: &\ (\ 9.9793,\ 3.7706,\ 9.2417) \end{aligned}$$

$$\begin{aligned} &New\ GPFU\ No.6\ center: \quad (\ 9.9646, 10.0510, 16.9319) \\ &New\ GPFU\ No.7\ center: \quad (\ 9.9629,\ 9.9948,\ 2.7820) \end{aligned} \tag{2.12}$$

The new shape matrices (inverse of the covariance matrices) of the GPFUs are:

$$New\ GPFU\ No.1\ shape\ matrix: \begin{bmatrix} 0.1422 & 0.0024 & 0.0022 \\ 0.0024 & 0.1404 & 0.0034 \\ 0.0022 & 0.0034 & 0.1193 \end{bmatrix} \tag{2.13}$$

$$New\ GPFU\ No.2\ shape\ matrix: \begin{bmatrix} 0.1373 & 0.0036 & 0.0076 \\ 0.0036 & 0.0528 & 0.0047 \\ 0.0076 & 0.0047 & 0.0839 \end{bmatrix} \tag{2.14}$$

$$New\ GPFU\ No.3\ shape\ matrix: \begin{bmatrix} 0.1428 & 0.0074 & 0.0086 \\ 0.0074 & 0.0474 & 0.0088 \\ 0.0086 & 0.0088 & 0.0665 \end{bmatrix} \tag{2.15}$$

$$New\ GPFU\ No.4\ shape\ matrix: \begin{bmatrix} 0.0537 & -0.0062 & -0.0032 \\ -0.0062 & 0.1311 & 0.0148 \\ -0.0032 & 0.0148 & 0.0354 \end{bmatrix} \tag{2.16}$$

$$New\ GPFU\ No.5\ shape\ matrix: \begin{bmatrix} 0.0607 & -0.0114 & -0.0005 \\ -0.0114 & 0.1424 & -0.0076 \\ -0.0005 & -0.0076 & 0.0336 \end{bmatrix} \tag{2.17}$$

$$New\ GPFU\ No.6\ shape\ matrix: \begin{bmatrix} -0.0062 & 0.0006 & -0.0005 \\ 0.0006 & -0.0042 & 0.0013 \\ -0.0005 & 0.0013 & -0.0034 \end{bmatrix} \tag{2.18}$$

$$New\ GPFU\ No.7\ shape\ matrix: \begin{bmatrix} -0.0042 & 0.0005 & -0.0034 \\ 0.0005 & -0.0012 & 0.0020 \\ -0.0034 & 0.0020 & 0.0608 \end{bmatrix} \tag{2.19}$$

It is noted that some of the diagonal elements of the shape matrices for GPFUs 6 and 7 have negative values. This is not compatible to theory which requires them positive. Their small values here indicate numerical computation effects.

The above results established a range of nominal feature values and a corresponding learning rate constant value set that exhibits good performance relative to convergence stability of the training algorithm.

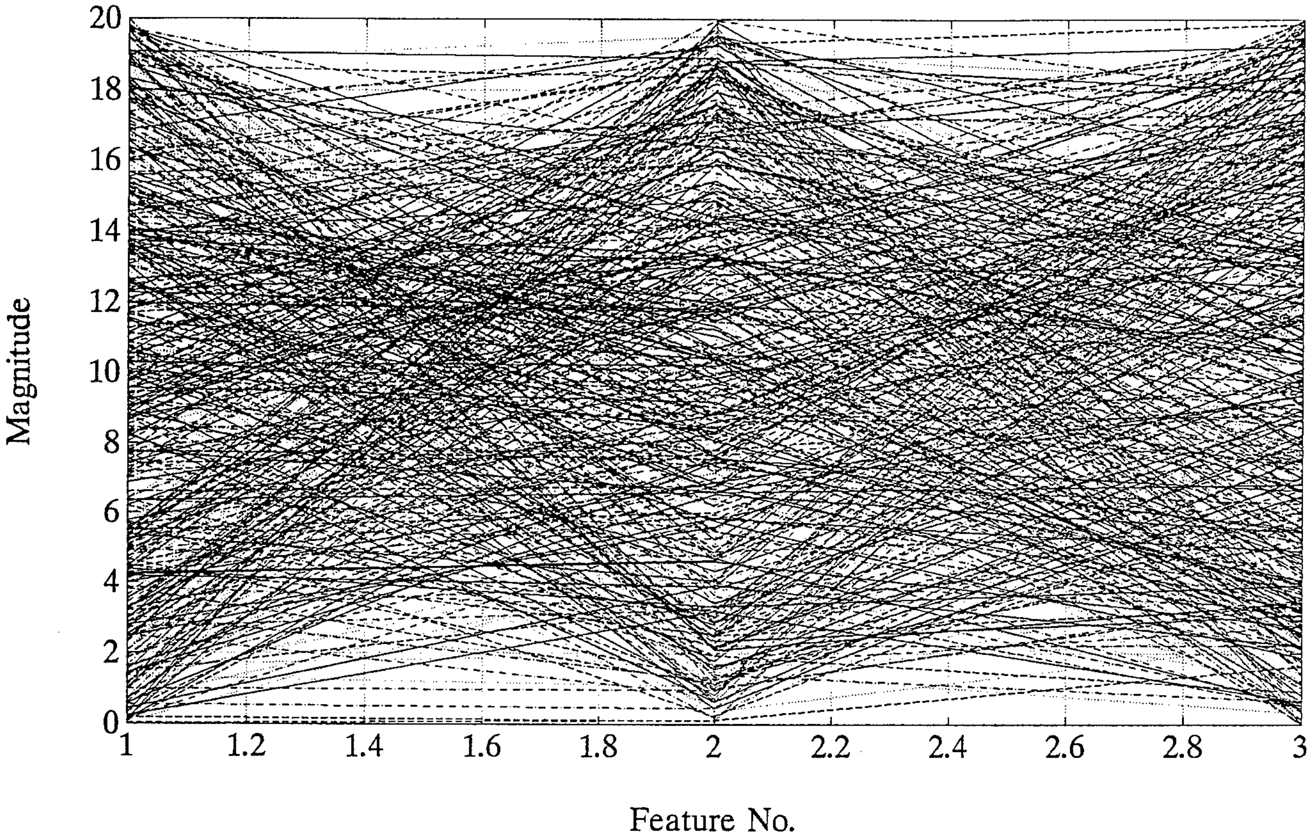

Figure 2.1: Feature Vectors plotted as Waveforms.

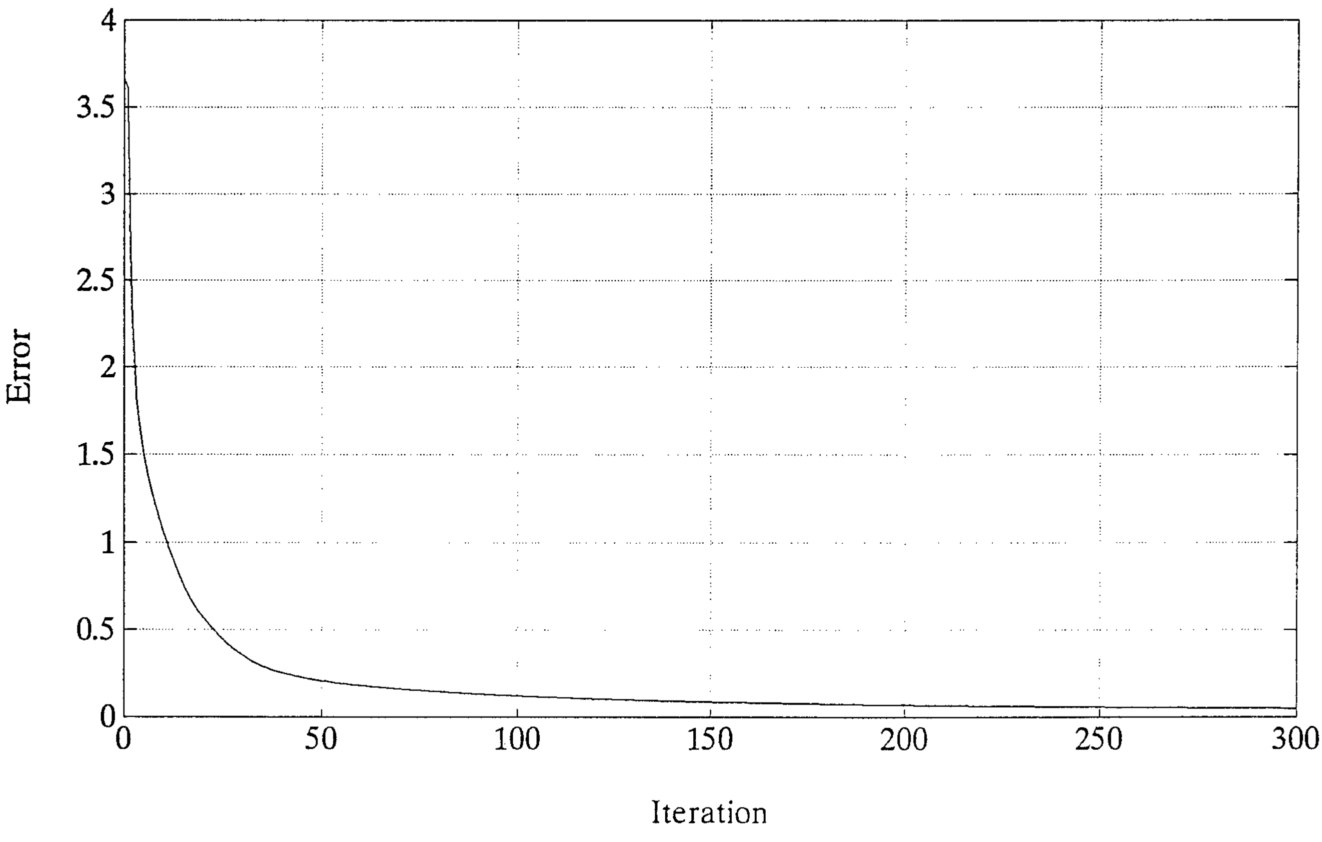

Figure 2.2: Learning Error Iteration History.

Chapter 3

Classification

In a classification problem the first act is the definition of classes that we want to partition our problem into. The next step is to train the classifier to yield the correct mapping from feature vectors to output classes for a set of data for which this relation is known from past experience. Thus, this classification training phase adjusts the parameters of the classifier so that it properly performs in the desired manner (i.e., high correct classification rate). The classification technique represented by the GPFN assigns to a given class a distinct output value. Thus for a multiclass problem we would choose a different integer value (say, 1,2,3, etc.) for each class. In the following we show the performance of the GPFN for a two class problem. We selected a two-dimensional feature vector so as to be able to illustrate geometrically our results. The methodology is directly extendable to any dimensions.

3.1 Class A

For Class A we first considered a nominal two dimensional feature vector of two elements: (10,10). We then generated 100 feature vectors by perturbing the nominal vector with a random value in the range -5 to 5. The 100 thus created feature vectors are shown in Figure 3.1. The feature vectors are plotted as waveforms for illustration purposes. The elements of the feature vectors are the ordinate values corresponding to the integer abscissa

values 1 and 2. The mean and standard deviation for each feature for the 100 feature vectors of Class A are:

$$\begin{aligned}
&Feature\ 1: Mean: 10.1588 \quad Standard\ Deviation:\ 2.9031 \\
&Feature\ 2: Mean: 10.0947 \quad Standard\ Deviation:\ 2.9008
\end{aligned} \tag{3.1}$$

Since there are two features there are 5 GPFUs centered at the mean value of the distribution and at one sigma distances from the mean in each direction for each feature axis.

The GPFUs are initially assigned unit amplitudes:

$$\begin{aligned}
&GPFU\ No.1\ amplitude:\ 1 \\
&GPFU\ No.2\ amplitude:\ 1 \\
&GPFU\ No.3\ amplitude:\ 1 \\
&GPFU\ No.4\ amplitude:\ 1 \\
&GPFU\ No.5\ amplitude:\ 1
\end{aligned} \tag{3.2}$$

and are centered at:

$$\begin{aligned}
&GPFU\ No.1\ center:\ (10.1588, 10.0947) \\
&GPFU\ No.2\ center:\ (13.0619, 10.0947) \\
&GPFU\ No.3\ center:\ (\ 7.2558, 10.0947) \\
&GPFU\ No.4\ center:\ (10.1588, 12.9955) \\
&GPFU\ No.5\ center:\ (10.1588,\ 7.1938)
\end{aligned} \tag{3.3}$$

with their sum shown in Figure 3.2.

The shape matrices (inverse of the covariance matrices) of the GPFUs are initially set equal to a diagonal matrix:

$$\begin{bmatrix} 0.4746 & 0 \\ 0 & 0.4753 \end{bmatrix} \tag{3.4}$$

with each diagonal element being equal to the inverse of 0.25 times the variance of the corresponding feature leading to a more compact GPFUs configuration.

We next trained the network for 1000 iterations. The resulting error time history is shown in Figure 3.3. It is seen that significant improvement results from the learning phase. The initial error (i.e. the square of the difference between the desired GPFN value of 1 and the actual output) is 0.3225. After 1000 iterations the error has dropped to a value of 0.0018. The learning phase has resulted into the amplitudes of the GPFUs having the values:

$$\begin{aligned} New\ GPFU\ No.1\ amplitude: &\ 0.9062 \\ New\ GPFU\ No.2\ amplitude: &\ 0.9114 \\ New\ GPFU\ No.3\ amplitude: &\ 0.8753 \\ New\ GPFU\ No.4\ amplitude: &\ 0.8929 \\ New\ GPFU\ No.5\ amplitude: &\ 1.0128 \end{aligned} \tag{3.5}$$

The mean vectors of the GPFUs have shifted to new positions as follows:

$$\begin{aligned} New\ GPFU\ No.1\ center: &\ (10.2671, 10.4148) \\ New\ GPFU\ No.2\ center: &\ (14.2591, 10.3745) \\ New\ GPFU\ No.3\ center: &\ (\ 6.0280,\ 9.1429) \\ New\ GPFU\ No.4\ center: &\ (10.5719, 14.4891) \\ New\ GPFU\ No.5\ center: &\ (11.8913,\ 5.7831) \end{aligned} \tag{3.6}$$

The new shape matrices (inverse of the covariance matrices) of the GPFUs are:

$$New\ GPFU\ No.1\ shape\ matrix: \begin{bmatrix} 0.3550 & 0.0111 \\ 0.0111 & 0.3528 \end{bmatrix} \tag{3.7}$$

$$New\ GPFU\ No.2\ shape\ matrix: \begin{bmatrix} 0.3635 & 0.0350 \\ 0.0350 & 0.1692 \end{bmatrix} \tag{3.8}$$

$$New\ GPFU\ No.3\ shape\ matrix: \begin{bmatrix} 0.3155 & -0.0220 \\ -0.0220 & 0.0973 \end{bmatrix} \tag{3.9}$$

$$New\ GPFU\ No.4\ shape\ matrix: \begin{bmatrix} 0.0186 & -0.0366 \\ -0.0366 & 0.3461 \end{bmatrix} \tag{3.10}$$

$$New\ GPFU\ No.5\ shape\ matrix: \begin{bmatrix} 0.0503 & 0.0218 \\ 0.0218 & 0.2542 \end{bmatrix} \tag{3.11}$$

The sum of the GPFUs under their new configuration, following the training phase, is shown in Figure 3.4.

3.2 Class B

For Class B we first considered a nominal two dimensional feature vector of two elements: (20,20). We then generated 100 feature vectors by perturbing the nominal vector with a random value in the range -5 to 5. The 100 thus created feature vectors are shown in Figure 3.5. The feature vectors are plotted as waveforms for illustration purposes. The elements of the feature vectors are the ordinate values corresponding to the integer abscissa values 1 and 2. The mean and standard deviation for each feature for the 100 feature vectors of Class B are:

$$\begin{aligned} &Feature\ 1 : Mean : 20.1588\ \ Standard\ Deviation :\ \ 2.9031 \\ &Feature\ 2 : Mean : 20.0947\ \ Standard\ Deviation :\ \ 2.9008 \end{aligned} \tag{3.12}$$

Since there are two features there are 5 GPFUs centered at the mean value of the distribution and at one sigma distances from the mean in each direction for each feature axis.

The initial amplitudes of the GPFUs are equal to 2:

$$\begin{aligned} &GPFU\ No.1\ amplitude :\ \ 2 \\ &GPFU\ No.2\ amplitude :\ \ 2 \\ &GPFU\ No.3\ amplitude :\ \ 2 \\ &GPFU\ No.4\ amplitude :\ \ 2 \\ &GPFU\ No.5\ amplitude :\ \ 2 \end{aligned} \tag{3.13}$$

and are centered at:

$$\begin{aligned} &GPFU\ No.1\ center :\ \ (20.1588, 20.0947) \\ &GPFU\ No.2\ center :\ \ (23.0619, 20.0947) \\ &GPFU\ No.3\ center :\ \ (17.2558, 20.0947) \\ &GPFU\ No.4\ center :\ \ (20.1588, 22.9955) \\ &GPFU\ No.5\ center :\ \ (20.1588, 17.1938) \end{aligned} \tag{3.14}$$

with their sum shown in Figure 3.6.

The shape matrices (inverse of the covariance matrices) of the GPFUs are initially set equal to a diagonal matrix

$$\begin{bmatrix} 0.4746 & 0 \\ 0 & 0.4753 \end{bmatrix} \tag{3.15}$$

with each diagonal element being equal to the inverse of 0.25 times the variance of the corresponding feature leading to a more compact GPFUs configuration.

We next trained the network for 1000 iterations. The resulting error time history is shown in Figure 3.7. It is seen that significant improvement results from the learning phase. The initial error (i.e. the square of the difference between the desired GPFN value of 2 and the actual output) is 1.2413. After 1000 iterations the error has dropped to a value of 0.0008. The learning phase has resulted into the amplitudes of the GPFUs having the values:

$$\begin{aligned} &New\ GPFU\ No.1\ amplitude: && 1.2888 \\ &New\ GPFU\ No.2\ amplitude: && 1.4253 \\ &New\ GPFU\ No.3\ amplitude: && 1.2908 \\ &New\ GPFU\ No.4\ amplitude: && 1.8088 \\ &New\ GPFU\ No.5\ amplitude: && 1.8982 \end{aligned} \tag{3.16}$$

The mean vectors of the GPFUs shifting to new positions as follows:

$$\begin{aligned} &New\ GPFU\ No.1\ center: && (20.4727, 20.2604) \\ &New\ GPFU\ No.2\ center: && (24.5723, 20, 5599) \\ &New\ GPFU\ No.3\ center: && (19.2209, 24.8075) \\ &New\ GPFU\ No.4\ center: && (19.2209, 24.8075) \\ &New\ GPFU\ No.5\ center: && (22.0443, 15.4032) \end{aligned} \tag{3.17}$$

The new shape matrices (inverse of the covariance matrices) of the GPFUs are:

$$New\ GPFU\ No.1\ shape\ matrix : \begin{bmatrix} 0.2858 & 0.0066 \\ 0.0066 & 0.2293 \end{bmatrix} \tag{3.18}$$

$$New\ GPFU\ No.2\ shape\ matrix : \begin{bmatrix} 0.2901 & 0.0240 \\ 0.0240 & 0.1851 \end{bmatrix} \tag{3.19}$$

$$New\ GPFU\ No.3\ shape\ matrix : \begin{bmatrix} 0.26375 & -0.0206 \\ -0.0206 & 0.1945 \end{bmatrix} \tag{3.20}$$

$$New\ GPFU\ No.4\ shape\ matrix : \begin{bmatrix} 0.0036 & -0.0236 \\ -0.0236 & 0.1796 \end{bmatrix} \tag{3.21}$$

$$New\ GPFU\ No.5\ shape\ matrix : \begin{bmatrix} 0.0057 & -0.0006 \\ -0.0006 & 0.1489 \end{bmatrix} \tag{3.22}$$

The sum of the GPFUs under their new configuration, following the training phase, is shown in Figure 3.8.

3.3 Class A versus B

The classification performance for the set of data representing Classes A and B is established as follows. The training phase of the classifier created two sets of GPFNs. One for Class A and one for Class B. A data point that belongs to Class A must ideally yield a value of 1 while a data point that belongs to Class B must yield the value 2. Figure 3.9 illustrates the superposition of the GPFN output values for the two classes. Figure 3.9 is thus the superposition of Figure 3.4 and Figure 3.8.

There are 200 data points to consider, 100 from Class A and 100 from Class B. Each point is fed to the GPFN corresponding to Class A and the GPFN corresponding to Class B.

Two responses are thus noted. Next, the percent deviation of the actual response from the desired response (the desired response is 1 for Class A and 2 for Class B) is calculated and the data point is assigned to the class with the smallest percent deviation. The results for the 200 points are given in Table 3.1. Column (1) of the Table lists the data point No., column (2) the known correct classification, column (3) the calculated classification, column (4) the response of the Class A GPFN, column (5) the percent error resulting from the Class A GPFN response, column (6) the response of the Class B GPFN and column (7) the percent error resulting from the Class B response. Thus, as an example, let us take data point 1. It belongs to class 1 (which is the same as class A) and has been correctly assigned to Class A because its response to the Class A GPFN is 0.9735, representing a 2.6478 percent error from the ideal value of 1, while its response to the Class B GPFN is 0.2048, representing an 89.7599 percent error from the ideal value of 2. Comparing columns (1) and (2) of Table 3.1 we note that all data poins have been correctly classified. This is not totally surprising because the data distributions from the two classes are nonoverlapping. The objects from Class A have a nominal center at (10,10) with dispersions from 5 to 15 in each feature while the objects from Class B have a nominal center at (20,20) with dispersions from 15 to 25 in each feature. Thus our results establish the capability of the GPFNs to tightly encode the data distributions.

To evaluate the GPFNs performance under overlapping conditions we next generated a new Class B (which we now call BB) with mean vector at (17,17) and dispersions from 12 to 22 in each feature. The data are shown in Figure 3.10. The initial GPFN distribution is shown in Figure 3.11 and the training error time history in Figure 3.12. The new GPFN configuration, following training, is shown in Figure 3.13. Figure 3.14 represents the superposition of the GPFNs for Classes A and BB following training. The classification results are shown in Table 3.2. It is now noted that 8 class A data points and 4 Class BB data points have being misclassified. The classifier thus yields a 94 % correct classification rate.

The optimum classifier design involves a training phase and a testing phase. Having defined

the features that constitute the feature vector and having established the number of classes that we want to partition our problem into, the first step is to train the classifier to yield the correct mapping from feature vectors to output classes for a set of data for which this relation is known from past experience. Thus, this classification training phase adjusts the parameters of the classifier so that it properly performs in the desired manner. Following this training phase the classifier is to be used as a future predictive tool to yield the right answers for feature vectors for which the classification outcome is not known. This is the, so called, testing phase of the classifier. It represents its generalization capability. In an intuitive sense, one desires that the training phase consists of data sufficient to capture the statistical essence in both depth and breadth of the problem under consideration. In the examples presented here we did not consider a testing phase because we utilize a computer simulation based on a known probabilistic data generation mechanism. Thus, sets of data so generated inherently possess similar statistical characteristics.

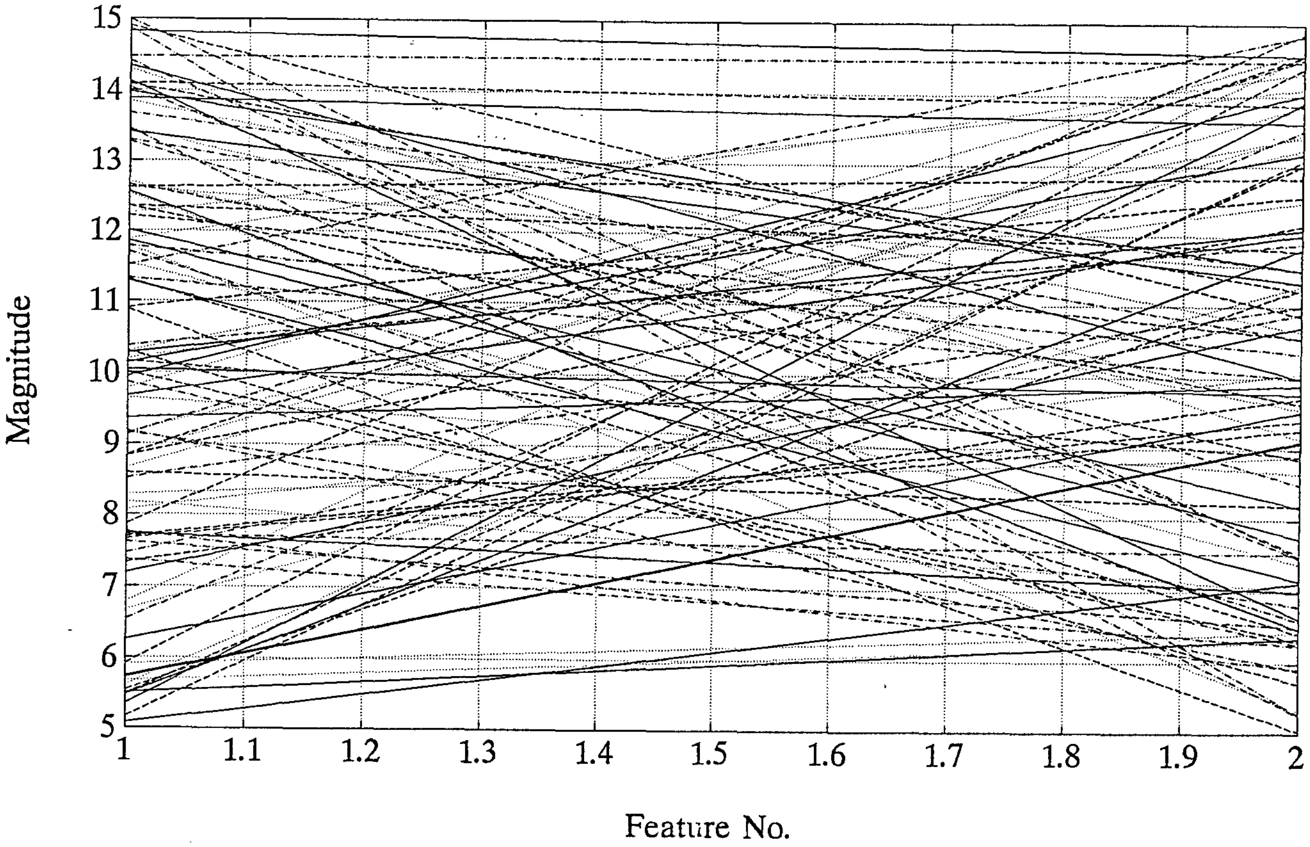

Figure 3.1: Class A feature vectors plotted as waveforms.

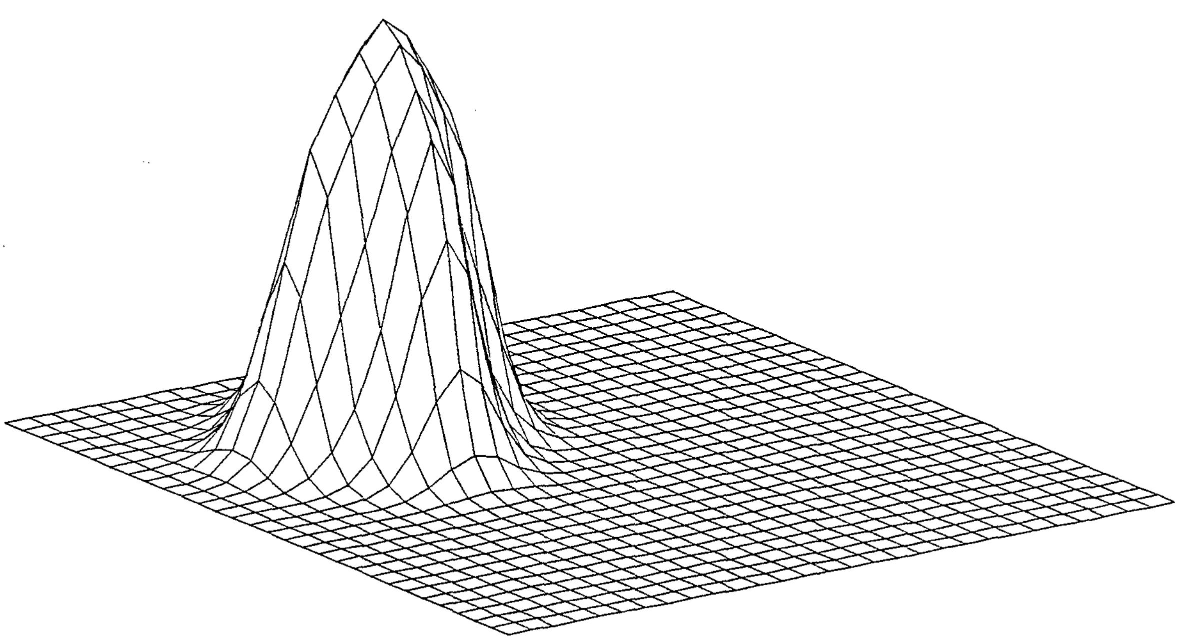

Figure 3.2: Initial Class A GPFUs distribution.

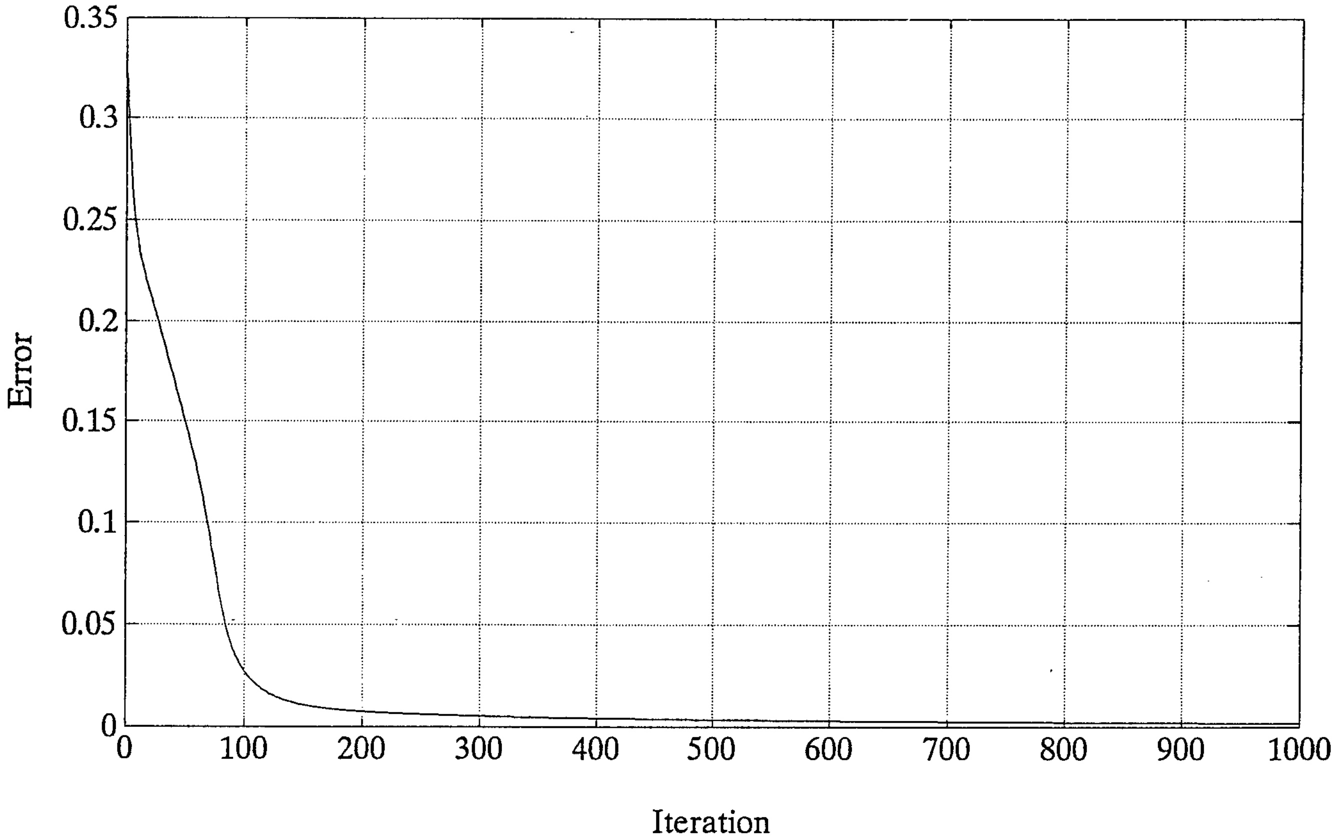

Figure 3.3: Class A learning error iteration history.

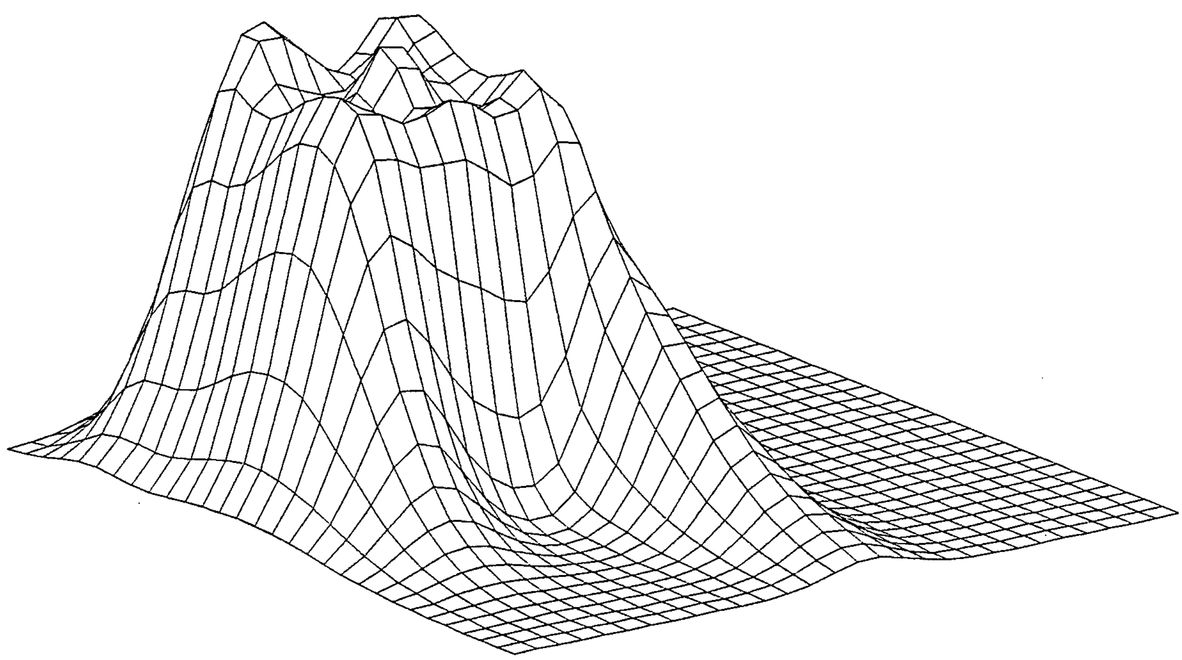

Figure 3.4: Class A GPFUs distribution following training.

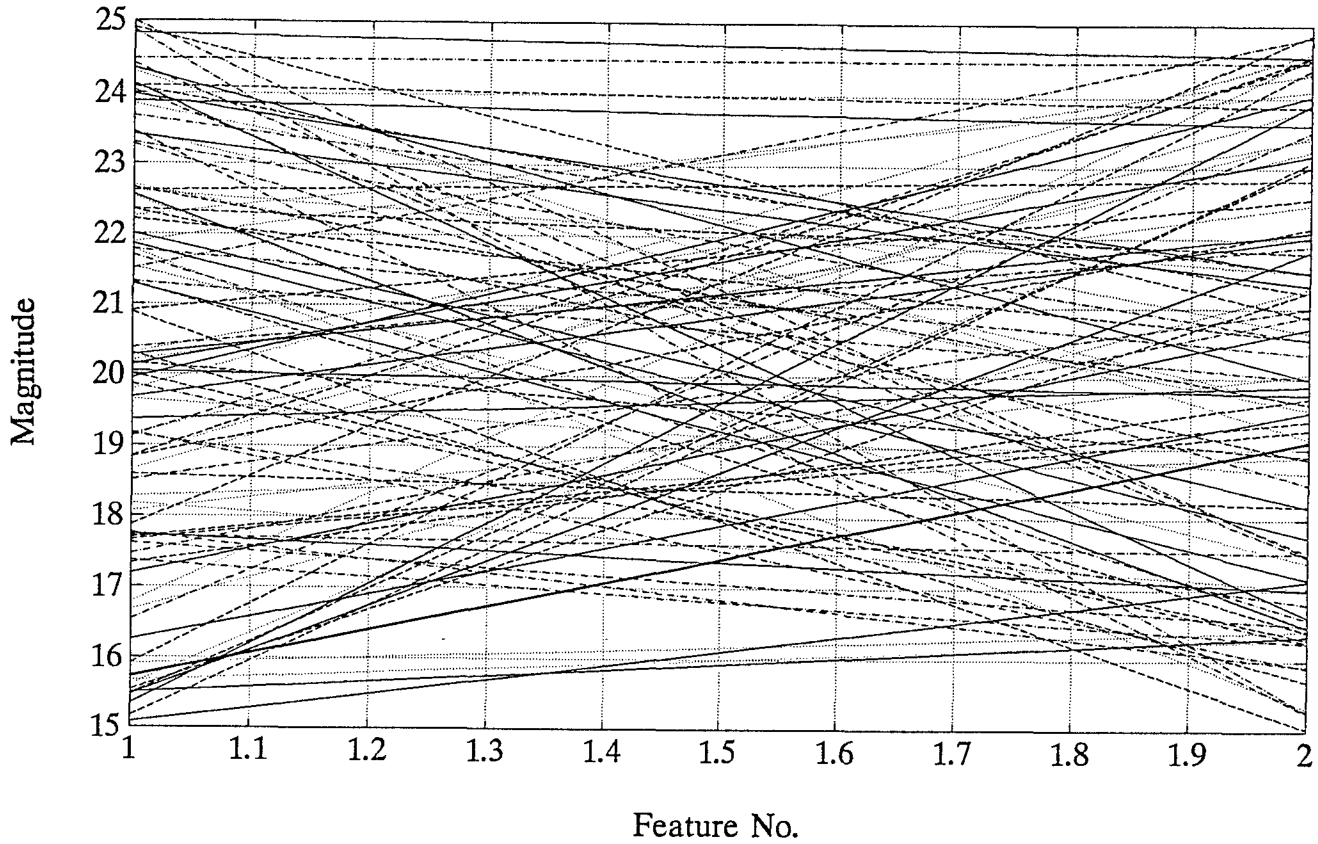

Figure 3.5: Class B feature vectors plotted as waveforms.

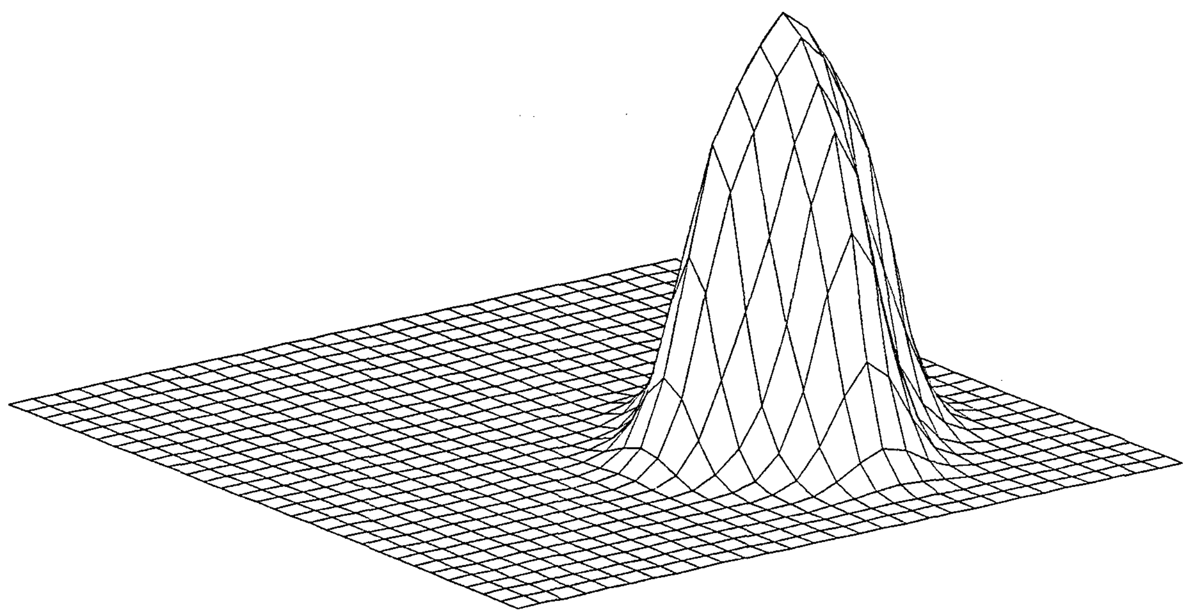

Figure 3.6: Initial Class B GPFUs distribution.

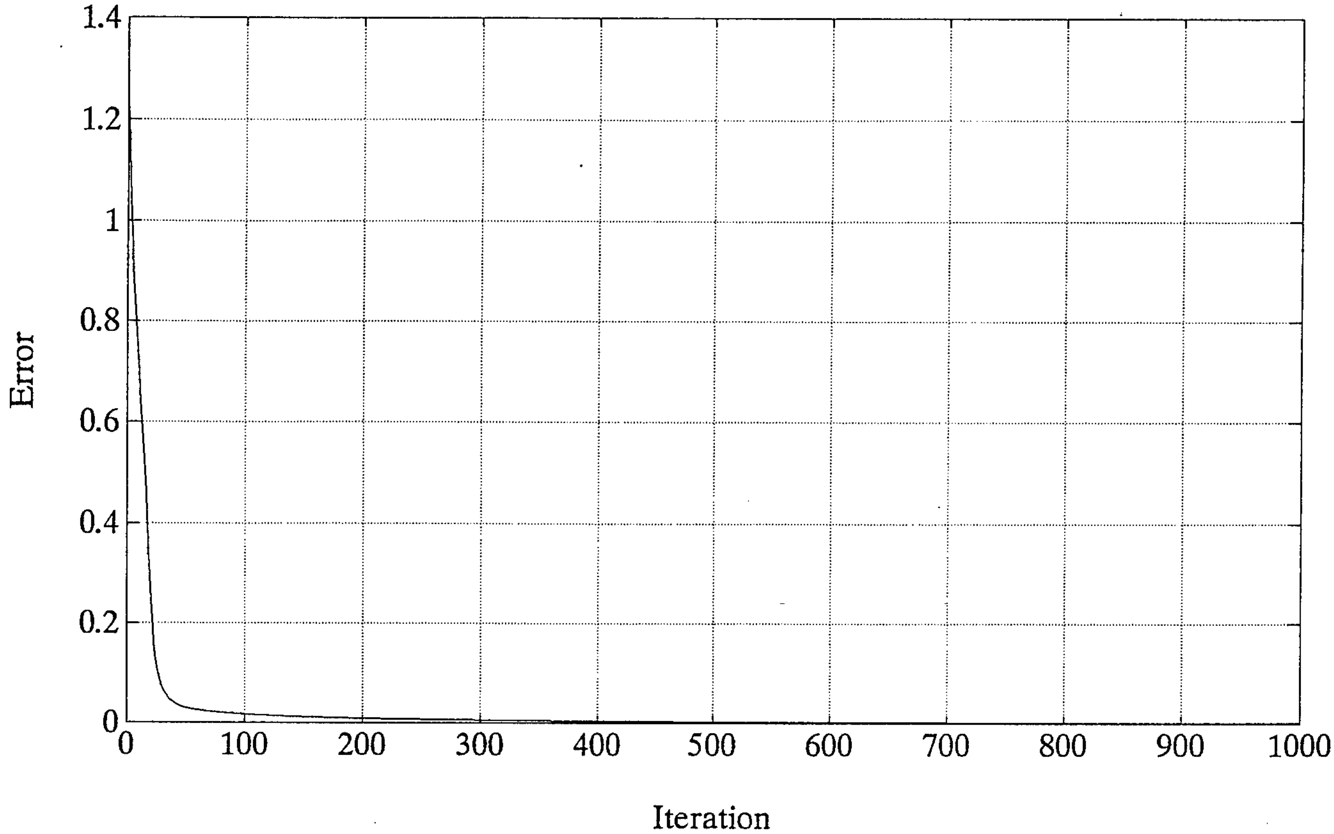

Figure 3.7: Class B learning error iteration history.

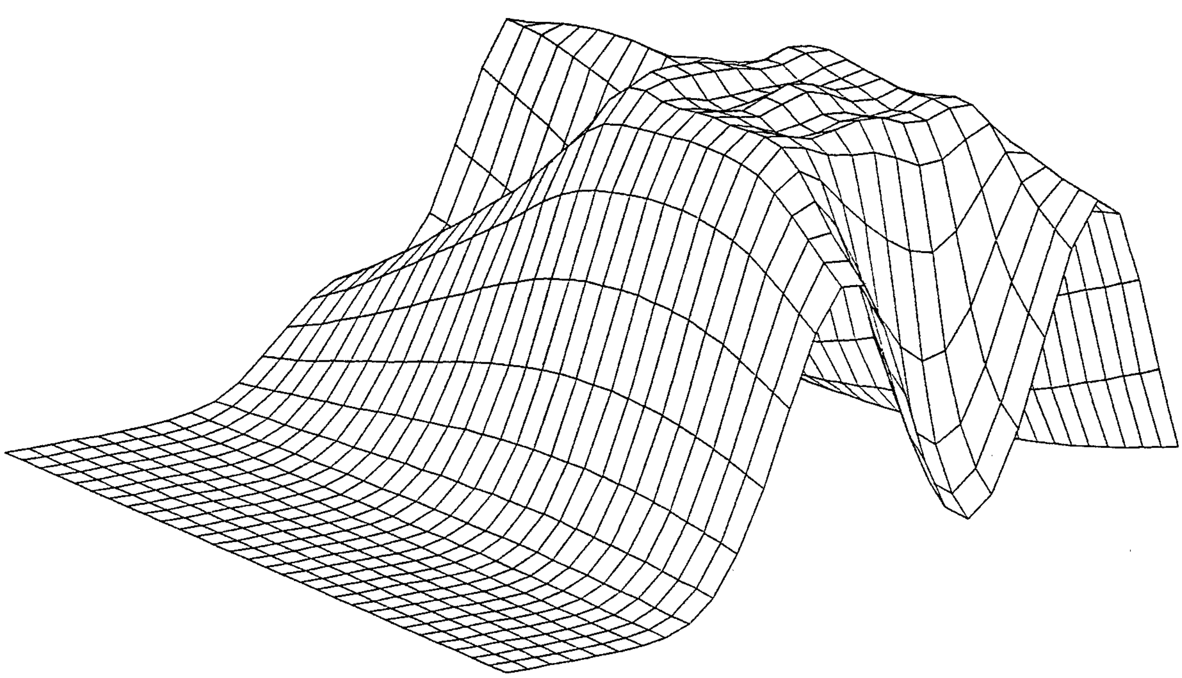

Figure 3.8: Class B GPFUs distribution following training.

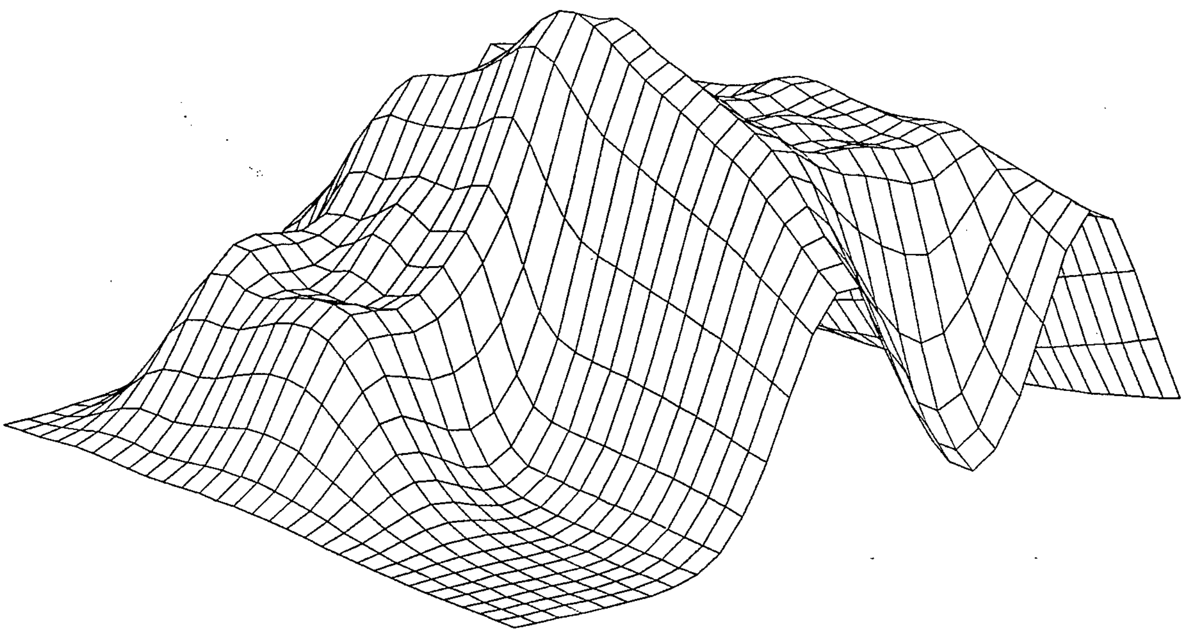

Figure 3.9: Superposition of Class A and Class B GPFUs distributions following training.

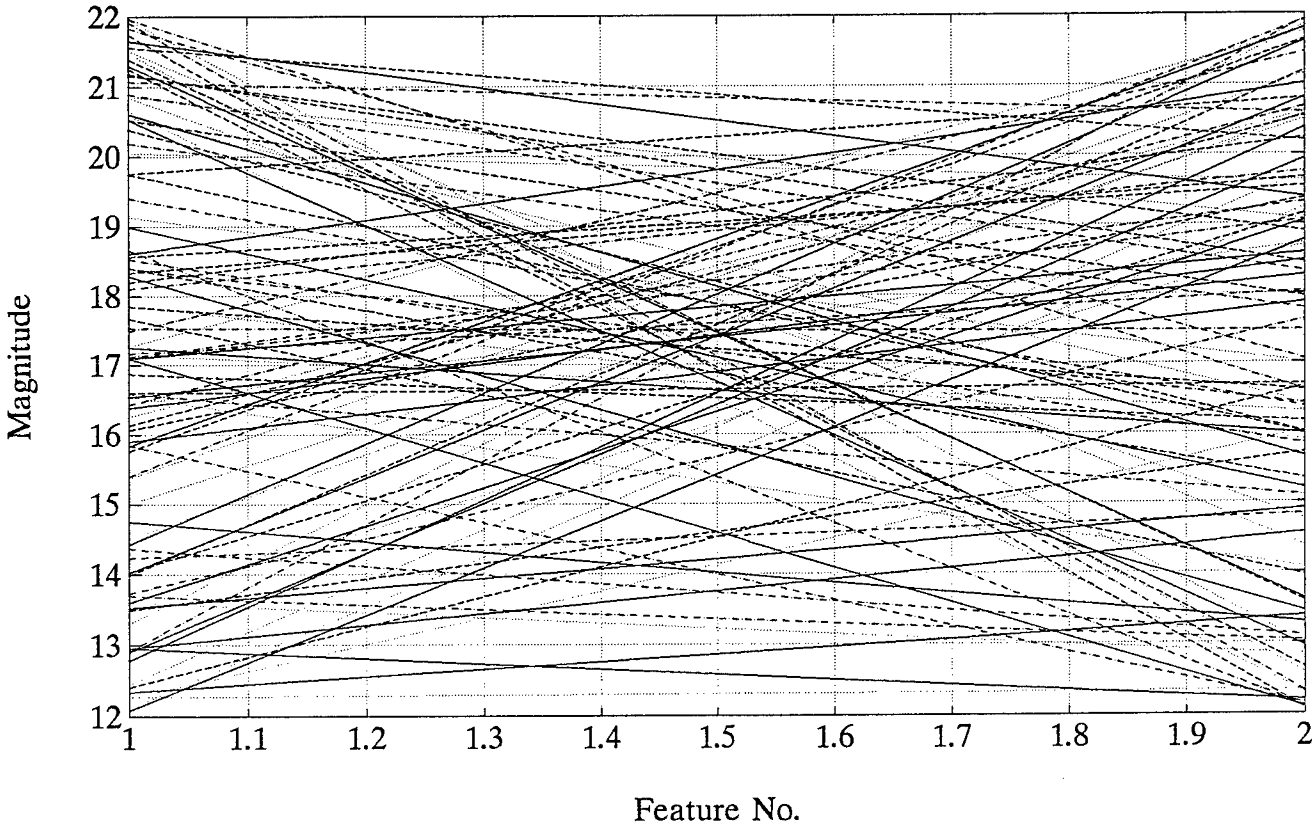

Figure 3.10: Class BB feature vectors plotted as waveforms.

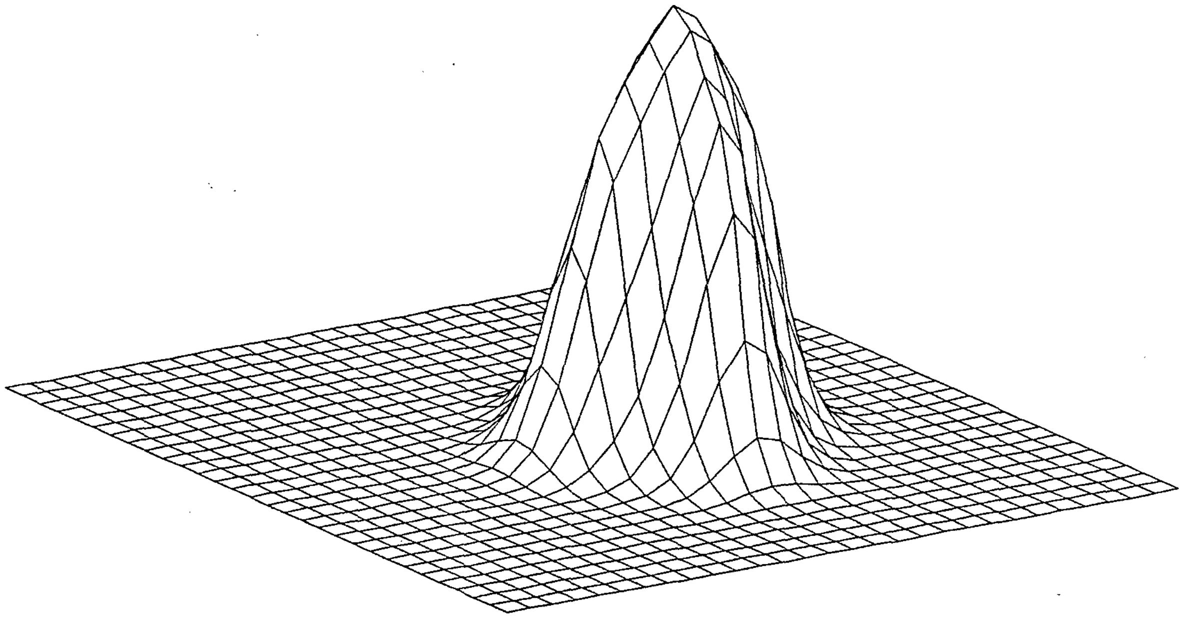

Figure 3.11: Initial Class BB GPFUs Distribution.

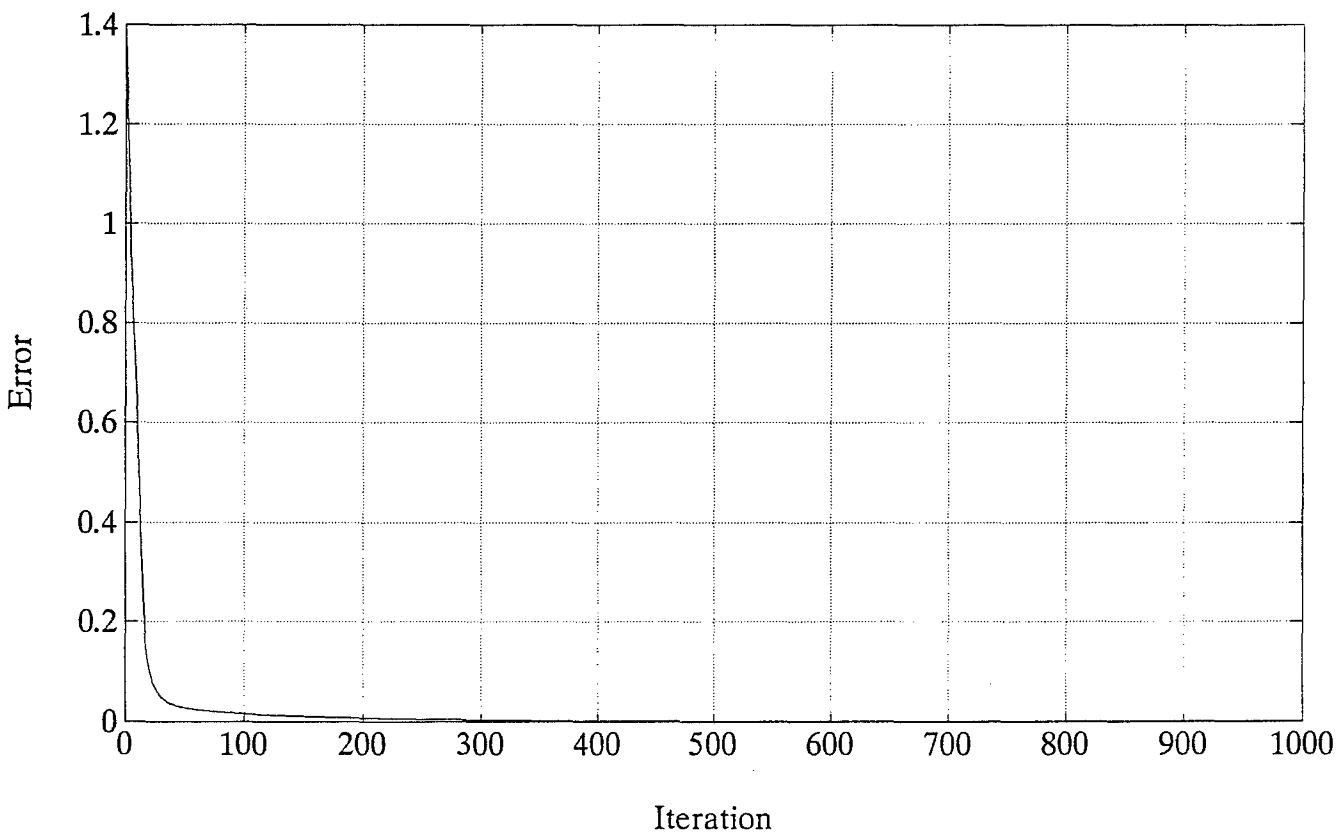

Figure 3.12: Class BB Learning Error Iteration History.

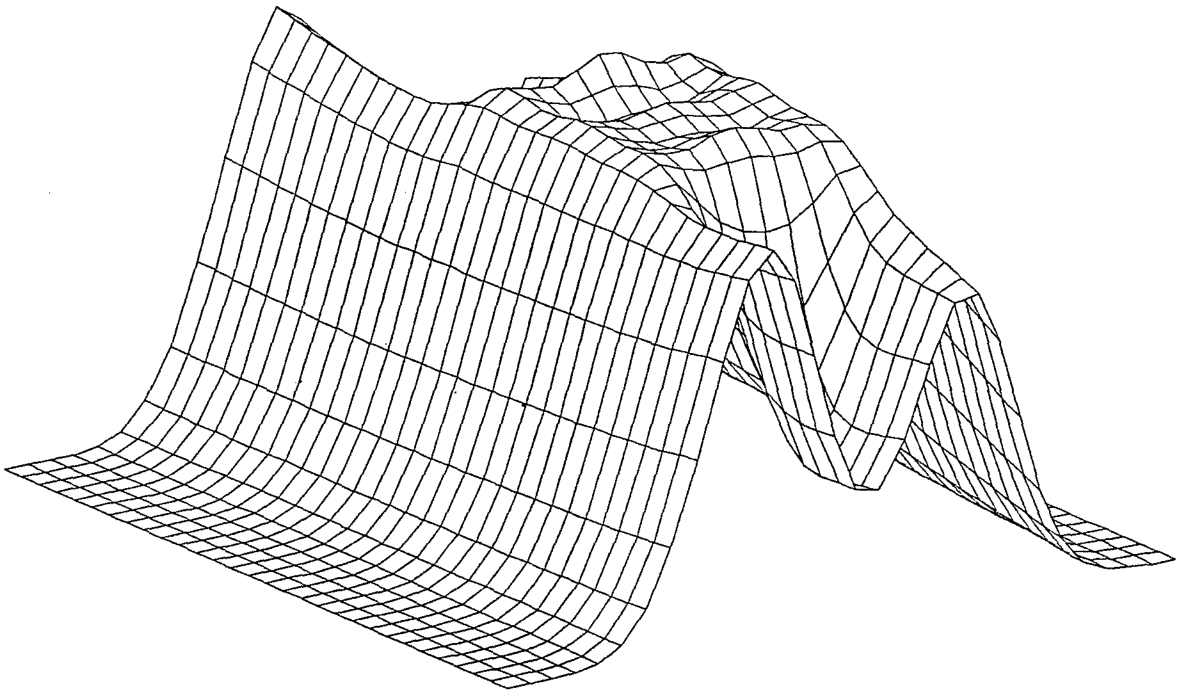

Figure 3.13: Class BB GPFUs distribution following training.

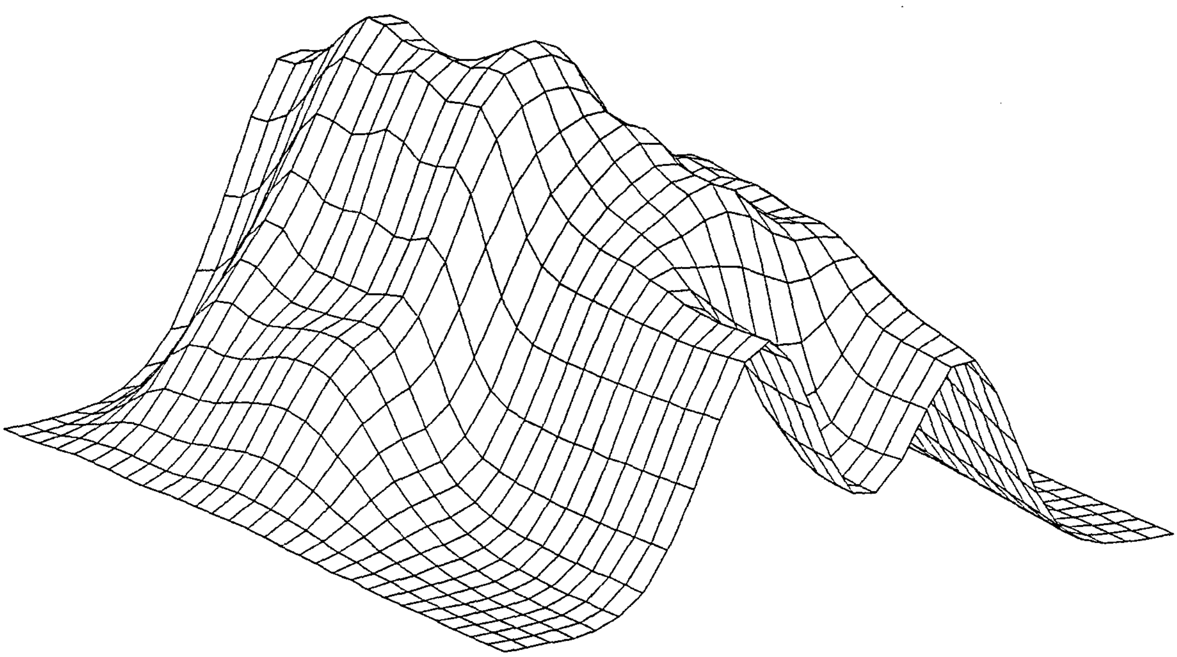

Figure 3.14: Superposition of Class A and Class BB GPFUs distributions following training.

Table 3.1: Class A versus Class B Classification Results.

Column (1): Object No.

Column (2): Correct Class.

Column (3): Assigned Class.

Column (4): Response to Class A assignment.

Column (5): Percent error corresponding to Class A assignment.

Column (6): Response to Class B assignment.

Column (7): Percent error corresponding to Class B assignment.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.0000	1.0000	1.0000	0.9735	2.6478	0.2048	89.7599
2.0000	1.0000	1.0000	1.0242	2.4229	0.5809	70.9548
3.0000	1.0000	1.0000	1.0163	1.6265	0.0008	99.9610
4.0000	1.0000	1.0000	1.0156	1.5600	0.2154	89.2289
5.0000	1.0000	1.0000	1.0323	3.2344	0.1845	90.7753
6.0000	1.0000	1.0000	0.9842	1.5804	1.1008	44.9595
7.0000	1.0000	1.0000	0.9752	2.4819	0.6869	65.6545
8.0000	1.0000	1.0000	1.0349	3.4871	0.2701	86.4962
9.0000	1.0000	1.0000	0.9833	1.6732	0.7353	63.2335
10.0000	1.0000	1.0000	0.9051	9.4874	0.2504	87.4780
11.0000	1.0000	1.0000	0.9925	0.7459	0.9682	51.5916
12.0000	1.0000	1.0000	1.0464	4.6360	0.0046	99.7709
13.0000	1.0000	1.0000	0.9566	4.3396	0.0055	99.7246
14.0000	1.0000	1.0000	0.9798	2.0199	0.5364	73.1800
15.0000	1.0000	1.0000	0.9902	0.9841	0.0020	99.8991
16.0000	1.0000	1.0000	0.9541	4.5949	0.0015	99.9228
17.0000	1.0000	1.0000	0.9396	6.0350	0.0188	99.0625
18.0000	1.0000	1.0000	0.9205	7.9473	0.0005	99.9765
19.0000	1.0000	1.0000	1.0200	1.9994	0.0889	95.5565
20.0000	1.0000	1.0000	1.0160	1.6010	0.0008	99.9614
21.0000	1.0000	1.0000	0.9932	0.6826	0.5790	71.0487
22.0000	1.0000	1.0000	0.9724	2.7588	0.8445	57.7747
23.0000	1.0000	1.0000	0.9652	3.4750	0.0123	99.3842
24.0000	1.0000	1.0000	0.9739	2.6089	0.0051	99.7428
25.0000	1.0000	1.0000	0.9195	8.0548	0.0303	98.4829
26.0000	1.0000	1.0000	1.0231	2.3079	1.3719	31.4073
27.0000	1.0000	1.0000	0.9651	3.4871	0.4901	75.4971
28.0000	1.0000	1.0000	1.0375	3.7460	0.0031	99.8452
29.0000	1.0000	1.0000	0.9254	7.4617	0.3431	82.8430
30.0000	1.0000	1.0000	0.9305	6.9516	0.0620	96.8998
31.0000	1.0000	1.0000	0.9549	4.5131	0.0488	97.5607
32.0000	1.0000	1.0000	1.0504	5.0357	0.1604	91.9810
33.0000	1.0000	1.0000	1.0769	7.6882	0.0041	99.7962
34.0000	1.0000	1.0000	0.9008	9.9177	0.3630	81.8508
35.0000	1.0000	1.0000	1.0461	4.6100	0.8702	56.4907
36.0000	1.0000	1.0000	0.9625	3.7493	0.2374	88.1312
37.0000	1.0000	1.0000	0.9433	5.6715	1.6386	18.0719
38.0000	1.0000	1.0000	0.9933	0.6738	0.2586	87.0722
39.0000	1.0000	1.0000	1.0758	7.5810	0.0041	99.7948
40.0000	1.0000	1.0000	0.9682	3.1818	1.3670	31.6506
41.0000	1.0000	1.0000	1.0426	4.2570	0.0483	97.5875
42.0000	1.0000	1.0000	1.0470	4.7004	0.0035	99.8231
43.0000	1.0000	1.0000	1.0497	4.9667	0.2979	85.1036
44.0000	1.0000	1.0000	1.0429	4.2946	0.0110	99.4481
45.0000	1.0000	1.0000	1.0626	6.2618	0.1293	93.5375
46.0000	1.0000	1.0000	0.9782	2.1764	0.1284	93.5786
47.0000	1.0000	1.0000	0.9674	3.2592	1.1868	40.6596
48.0000	1.0000	1.0000	1.0288	2.8782	0.0022	99.8903
49.0000	1.0000	1.0000	1.0471	4.7062	0.0059	99.7058
50.0000	1.0000	1.0000	0.9424	5.7586	0.0249	98.7544

Table 3.1 (cont.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
51.0000	1.0000	1.0000	0.9867	1.3277	0.2844	85.7794
52.0000	1.0000	1.0000	0.9956	0.4372	0.0026	99.8677
53.0000	1.0000	1.0000	1.0054	0.5365	0.5211	73.9464
54.0000	1.0000	1.0000	1.0146	1.4565	0.4386	78.0707
55.0000	1.0000	1.0000	1.0419	4.1926	0.5896	70.5198
56.0000	1.0000	1.0000	1.0302	3.0243	0.0153	99.2366
57.0000	1.0000	1.0000	1.1007	10.0735	0.1225	93.8749
58.0000	1.0000	1.0000	0.9985	0.1492	0.0565	97.1767
59.0000	1.0000	1.0000	1.0078	0.7759	0.0064	99.6823
60.0000	1.0000	1.0000	0.9175	8.2482	0.0008	99.9583
61.0000	1.0000	1.0000	1.0122	1.2231	1.0935	45.3233
62.0000	1.0000	1.0000	0.9881	1.1919	0.0668	96.6583
63.0000	1.0000	1.0000	1.0397	3.9666	0.0024	99.8796
64.0000	1.0000	1.0000	0.9862	1.3767	1.5873	20.6372
65.0000	1.0000	1.0000	1.0428	4.2834	0.0490	97.5502
66.0000	1.0000	1.0000	1.0014	0.1398	0.0028	99.8582
67.0000	1.0000	1.0000	1.0438	4.3845	0.9768	51.1610
68.0000	1.0000	1.0000	0.9642	3.5759	0.0014	99.9291
69.0000	1.0000	1.0000	1.0605	6.0481	0.0053	99.7349
70.0000	1.0000	1.0000	0.9898	1.0219	0.0013	99.9374
71.0000	1.0000	1.0000	0.9624	3.7559	0.0431	97.8428
72.0000	1.0000	1.0000	1.0082	0.8167	0.0165	99.1765
73.0000	1.0000	1.0000	0.9698	3.0219	0.0021	99.8941
74.0000	1.0000	1.0000	0.9741	2.5883	0.9160	54.1976
75.0000	1.0000	1.0000	0.9759	2.4105	0.1194	94.0313
76.0000	1.0000	1.0000	0.9827	1.7292	0.0452	97.7415
77.0000	1.0000	1.0000	1.0521	5.2091	0.0748	96.2621
78.0000	1.0000	1.0000	0.9811	1.8875	0.5768	71.1606
79.0000	1.0000	1.0000	1.0275	2.7518	1.3135	34.3248
80.0000	1.0000	1.0000	1.0125	1.2487	0.5153	74.2357
81.0000	1.0000	1.0000	0.9860	1.4039	0.0090	99.5486
82.0000	1.0000	1.0000	0.9454	5.4598	0.5629	71.8569
83.0000	1.0000	1.0000	1.0426	4.2552	1.2908	35.4604
84.0000	1.0000	1.0000	0.9331	6.6905	0.0163	99.1867
85.0000	1.0000	1.0000	1.0295	2.9512	1.2667	36.6643
86.0000	1.0000	1.0000	0.9978	0.2215	0.0962	95.1899
87.0000	1.0000	1.0000	0.9673	3.2689	0.1351	93.2428
88.0000	1.0000	1.0000	0.9824	1.7612	0.2753	86.2343
89.0000	1.0000	1.0000	0.9858	1.4203	0.8882	55.5922
90.0000	1.0000	1.0000	0.9491	5.0859	0.7428	62.8587
91.0000	1.0000	1.0000	1.0304	3.0429	0.1314	93.4289
92.0000	1.0000	1.0000	0.9860	1.3980	1.1189	44.0567
93.0000	1.0000	1.0000	1.0152	1.5193	0.4551	77.2438
94.0000	1.0000	1.0000	0.9829	1.7131	0.0752	96.2416
95.0000	1.0000	1.0000	1.0166	1.6630	0.8485	57.5749
96.0000	1.0000	1.0000	1.0085	0.8489	0.5252	73.7423
97.0000	1.0000	1.0000	1.0017	0.1725	0.5361	73.1969
98.0000	1.0000	1.0000	0.9258	7.4243	1.0539	47.3045
99.0000	1.0000	1.0000	0.9930	0.6984	0.9321	53.3943
100.0000	1.0000	1.0000	1.1171	11.7059	0.7441	62.7950

(1)	(2)	(3)	(4)	(5)	(6)	(7)
101.0000	2.0000	2.0000	0.0033	99.6724	2.0122	0.6102
102.0000	2.0000	2.0000	0.0000	99.9988	2.0157	0.7863
103.0000	2.0000	2.0000	0.3466	65.3439	1.9898	0.5125
104.0000	2.0000	2.0000	0.0081	99.1870	2.0139	0.6935
105.0000	2.0000	2.0000	0.0131	98.6870	2.0320	1.5976
106.0000	2.0000	2.0000	0.0000	100.0000	1.9915	0.4266
107.0000	2.0000	2.0000	0.0001	99.9901	1.9904	0.4822
108.0000	2.0000	2.0000	0.0058	99.4178	2.0264	1.3186
109.0000	2.0000	2.0000	0.0000	99.9999	2.0087	0.4346
110.0000	2.0000	2.0000	0.0009	99.9089	1.9269	3.6547
111.0000	2.0000	2.0000	0.0000	99.9991	2.0034	0.1692
112.0000	2.0000	2.0000	0.3084	69.1573	2.0204	1.0190
113.0000	2.0000	2.0000	0.3565	64.3483	1.9338	3.3085
114.0000	2.0000	2.0000	0.0002	99.9812	1.9734	1.3283
115.0000	2.0000	2.0000	0.5806	41.9382	1.9950	0.2513
116.0000	2.0000	2.0000	0.4954	50.4613	1.9992	0.0422
117.0000	2.0000	2.0000	0.1673	83.2741	1.9638	1.8096
118.0000	2.0000	2.0000	0.3871	61.2886	1.9391	3.0434
119.0000	2.0000	2.0000	0.0378	96.2181	2.0144	0.7179
120.0000	2.0000	2.0000	0.2479	75.2130	1.9852	0.7394
121.0000	2.0000	2.0000	0.0002	99.9753	1.9989	0.0562
122.0000	2.0000	2.0000	0.0000	100.0000	1.9822	0.8897
123.0000	2.0000	2.0000	0.2169	78.3141	1.9848	0.7578
124.0000	2.0000	2.0000	0.3677	63.2326	1.9865	0.6736
125.0000	2.0000	2.0000	0.1179	88.2079	1.9526	2.3718
126.0000	2.0000	2.0000	0.0000	99.9996	2.0063	0.3161
127.0000	2.0000	2.0000	0.0010	99.9010	1.9710	1.4523
128.0000	2.0000	2.0000	0.4711	52.8906	2.0154	0.7676
129.0000	2.0000	2.0000	0.0003	99.9746	1.9361	3.1928
130.0000	2.0000	2.0000	0.0594	94.0585	1.9533	2.3355
131.0000	2.0000	2.0000	0.0633	93.6669	1.9496	2.5224
132.0000	2.0000	2.0000	0.0140	98.6031	2.0473	2.3648
133.0000	2.0000	2.0000	0.2834	71.6567	2.0246	1.2278
134.0000	2.0000	2.0000	0.0033	99.6736	1.9105	4.4756
135.0000	2.0000	2.0000	0.0000	99.9994	2.0225	1.1239
136.0000	2.0000	2.0000	0.0024	99.7643	1.9924	0.3797
137.0000	2.0000	2.0000	0.0000	99.9999	1.9707	1.4639
138.0000	2.0000	2.0000	0.0057	99.4297	1.9929	0.3548
139.0000	2.0000	2.0000	0.2843	71.5708	2.0241	1.2037
140.0000	2.0000	2.0000	0.0000	100.0000	1.9907	0.4662
141.0000	2.0000	2.0000	0.0434	95.6610	2.0139	0.6929
142.0000	2.0000	2.0000	0.3425	65.7528	2.0293	1.4636
143.0000	2.0000	2.0000	0.0049	99.5131	2.0464	2.3185
144.0000	2.0000	2.0000	0.2506	74.9382	1.9907	0.4628
145.0000	2.0000	2.0000	0.0160	98.3987	2.0218	1.0880
146.0000	2.0000	2.0000	0.0227	97.7281	1.9805	0.9730
147.0000	2.0000	2.0000	0.0000	100.0000	1.9910	0.4489
148.0000	2.0000	2.0000	0.5288	47.1202	2.0190	0.9477
149.0000	2.0000	2.0000	0.3610	63.9043	2.0031	0.1570
150.0000	2.0000	2.0000	0.1357	86.4277	1.9392	3.0382

Table 3.1 (cont.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
151.0000	2.0000	2.0000	0.0010	99.9038	2.0196	0.9809
152.0000	2.0000	2.0000	0.4240	57.6041	2.0170	0.8490
153.0000	2.0000	2.0000	0.0010	99.8966	2.0129	0.6452
154.0000	2.0000	2.0000	0.0018	99.8191	2.0219	1.0968
155.0000	2.0000	2.0000	0.0000	99.9988	2.0262	1.3088
156.0000	2.0000	2.0000	0.1537	84.6321	2.0095	0.4749
157.0000	2.0000	2.0000	0.0196	98.0439	2.0686	3.4287
158.0000	2.0000	2.0000	0.0621	93.7887	2.0290	1.4483
159.0000	2.0000	2.0000	0.3459	65.4054	1.9825	0.8771
160.0000	2.0000	2.0000	0.1816	81.8400	1.9699	1.5044
161.0000	2.0000	2.0000	0.0000	99.9998	2.0185	0.9263
162.0000	2.0000	2.0000	0.0374	96.2643	1.9901	0.4932
163.0000	2.0000	2.0000	0.5419	45.8082	2.0194	0.9696
164.0000	2.0000	2.0000	0.0000	99.9999	2.0017	0.0848
165.0000	2.0000	2.0000	0.0425	95.7518	2.0153	0.7626
166.0000	2.0000	2.0000	0.4123	58.7721	2.0196	0.9817
167.0000	2.0000	2.0000	0.0000	99.9998	2.0271	1.3566
168.0000	2.0000	2.0000	0.5677	43.2320	1.9968	0.1616
169.0000	2.0000	2.0000	0.2118	78.8174	2.0446	2.2291
170.0000	2.0000	2.0000	0.4423	55.7658	2.0124	0.6181
171.0000	2.0000	2.0000	0.0824	91.7628	1.9896	0.5197
172.0000	2.0000	2.0000	0.1418	85.8206	2.0081	0.4052
173.0000	2.0000	2.0000	0.5730	42.7027	1.9820	0.9010
174.0000	2.0000	2.0000	0.0001	99.9946	1.9657	1.7154
175.0000	2.0000	2.0000	0.0255	97.4535	1.9782	1.0914
176.0000	2.0000	2.0000	0.0798	92.0191	1.9664	1.6778
177.0000	2.0000	2.0000	0.0237	97.6332	2.0609	3.0450
178.0000	2.0000	2.0000	0.0000	99.9991	1.9923	0.3870
179.0000	2.0000	2.0000	0.0000	99.9999	2.0173	0.8641
180.0000	2.0000	2.0000	0.0010	99.8977	2.0161	0.8053
181.0000	2.0000	2.0000	0.2556	74.4428	2.0001	0.0035
182.0000	2.0000	2.0000	0.0006	99.9442	1.9564	2.1822
183.0000	2.0000	2.0000	0.0000	99.9998	2.0197	0.9851
184.0000	2.0000	2.0000	0.1318	86.8223	1.9783	1.0834
185.0000	2.0000	2.0000	0.0000	99.9992	2.0069	0.3432
186.0000	2.0000	2.0000	0.0190	98.1045	2.0185	0.9256
187.0000	2.0000	2.0000	0.0114	98.8634	1.9716	1.4224
188.0000	2.0000	2.0000	0.0021	99.7869	1.9675	1.6230
189.0000	2.0000	2.0000	0.0000	99.9983	1.9990	0.0507
190.0000	2.0000	2.0000	0.0001	99.9911	1.9725	1.3742
191.0000	2.0000	2.0000	0.0225	97.7537	2.0162	0.8092
192.0000	2.0000	2.0000	0.0000	100.0000	1.9969	0.1564
193.0000	2.0000	2.0000	0.0015	99.8533	2.0154	0.7684
194.0000	2.0000	2.0000	0.0312	96.8806	1.9885	0.5737
195.0000	2.0000	2.0000	0.0000	99.9988	2.0096	0.4786
196.0000	2.0000	2.0000	0.0004	99.9602	2.0071	0.3552
197.0000	2.0000	2.0000	0.0003	99.9724	1.9957	0.2158
198.0000	2.0000	2.0000	0.0000	100.0000	1.9318	3.4084
199.0000	2.0000	2.0000	0.0000	100.0000	1.9787	1.0628
200.0000	2.0000	2.0000	0.0000	99.9996	2.0651	3.2563

Table 3.2: Class A versus Class BB Classification Results.

Column (1): Object No.
Column (2): Correct Class.
Column (3): Assigned Class.
Column (4): Response to Class A assignment.
Column (5): Percent error corresponding to Class A assignment.
Column (6): Response to Class BB assignment.
Column (7): Percent error corresponding to Class BB assignment.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.0000	1.0000	1.0000	0.9735	2.6478	1.5940	20.3009
2.0000	1.0000	1.0000	1.0242	2.4229	1.8291	8.5434
3.0000	1.0000	1.0000	1.0163	1.6265	0.0396	98.0180
4.0000	1.0000	1.0000	1.0156	1.5600	1.4340	28.3005
5.0000	1.0000	1.0000	1.0323	3.2344	1.2766	36.1694
6.0000	1.0000	1.0000	0.9842	1.5804	1.2326	38.3709
7.0000	1.0000	1.0000	0.9752	2.4819	1.9015	4.9255
8.0000	1.0000	1.0000	1.0349	3.4871	1.5273	23.6356
9.0000	1.0000	1.0000	0.9833	1.6732	1.5220	23.8985
10.0000	1.0000	1.0000	0.9051	9.4874	1.7930	10.3488
11.0000	1.0000	1.0000	0.9925	0.7459	1.7622	11.8882
12.0000	1.0000	1.0000	1.0464	4.6360	0.1453	92.7370
13.0000	1.0000	1.0000	0.9566	4.3396	0.2274	88.6305
14.0000	1.0000	1.0000	0.9798	2.0199	1.8896	5.5190
15.0000	1.0000	1.0000	0.9902	0.9841	0.1073	94.6326
16.0000	1.0000	1.0000	0.9541	4.5949	0.0730	96.3493
17.0000	1.0000	1.0000	0.9396	6.0350	0.3748	81.2620
18.0000	1.0000	1.0000	0.9205	7.9473	0.0278	98.6102
19.0000	1.0000	1.0000	1.0200	1.9994	0.8896	55.5224
20.0000	1.0000	1.0000	1.0160	1.6010	0.0375	98.1243
21.0000	1.0000	1.0000	0.9932	0.6826	1.9033	4.8362
22.0000	1.0000	1.0000	0.9724	2.7588	1.2980	35.0989
23.0000	1.0000	1.0000	0.9652	3.4750	0.2877	85.6154
24.0000	1.0000	1.0000	0.9739	2.6089	0.1738	91.3119
25.0000	1.0000	1.0000	0.9195	8.0548	0.5052	74.7422
26.0000	1.0000	2.0000	1.0231	2.3079	1.9986	0.0703
27.0000	1.0000	1.0000	0.9651	3.4871	1.8674	6.6317
28.0000	1.0000	1.0000	1.0375	3.7460	0.1307	93.4675
29.0000	1.0000	2.0000	0.9254	7.4617	1.9106	4.4718
30.0000	1.0000	1.0000	0.9305	6.9516	0.7628	61.8604
31.0000	1.0000	1.0000	0.9549	4.5131	0.7673	61.6355
32.0000	1.0000	1.0000	1.0504	5.0357	1.2752	36.2409
33.0000	1.0000	1.0000	1.0769	7.6882	0.1303	93.4829
34.0000	1.0000	1.0000	0.9008	9.9177	1.6561	17.1967
35.0000	1.0000	1.0000	1.0461	4.6100	1.6619	16.9049
36.0000	1.0000	1.0000	0.9625	3.7493	1.6600	16.9977
37.0000	1.0000	2.0000	0.9433	5.6715	1.9198	4.0085
38.0000	1.0000	1.0000	0.9933	0.6738	1.5300	23.5012
39.0000	1.0000	1.0000	1.0758	7.5810	0.1311	93.4463
40.0000	1.0000	1.0000	0.9682	3.1818	1.7397	13.0128
41.0000	1.0000	1.0000	1.0426	4.2570	0.8630	56.8487
42.0000	1.0000	1.0000	1.0470	4.7004	0.1225	93.8738
43.0000	1.0000	1.0000	1.0497	4.9667	1.5704	21.4823
44.0000	1.0000	1.0000	1.0429	4.2946	0.3132	84.3397
45.0000	1.0000	1.0000	1.0626	6.2618	1.2217	38.9135
46.0000	1.0000	1.0000	0.9782	2.1764	1.1132	44.3410
47.0000	1.0000	1.0000	0.9674	3.2592	1.3198	34.0085
48.0000	1.0000	1.0000	1.0288	2.8782	0.1033	94.8359
49.0000	1.0000	1.0000	1.0471	4.7062	0.2041	89.7934
50.0000	1.0000	1.0000	0.9424	5.7586	0.5073	74.6350

Table 3.2 (cont.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
51.0000	1.0000	1.0000	0.9867	1.3277	1.7906	10.4712
52.0000	1.0000	1.0000	0.9956	0.4372	0.1051	94.7466
53.0000	1.0000	1.0000	1.0054	0.5365	1.8662	6.6920
54.0000	1.0000	1.0000	1.0146	1.4565	1.7789	11.0553
55.0000	1.0000	1.0000	1.0419	4.1926	1.8247	8.7626
56.0000	1.0000	1.0000	1.0302	3.0243	0.3099	84.5028
57.0000	1.0000	1.0000	1.1007	10.0735	1.1652	41.7377
58.0000	1.0000	1.0000	0.9985	0.1492	0.7738	61.3097
59.0000	1.0000	1.0000	1.0078	0.7759	0.2106	89.4724
60.0000	1.0000	1.0000	0.9175	8.2482	0.0387	98.0651
61.0000	1.0000	1.0000	1.0122	1.2231	1.5485	22.5768
62.0000	1.0000	1.0000	0.9881	1.1919	0.9381	53.0942
63.0000	1.0000	1.0000	1.0397	3.9666	0.1202	93.9905
64.0000	1.0000	1.0000	0.9862	1.3767	1.9413	2.9327
65.0000	1.0000	1.0000	1.0428	4.2834	0.8699	56.5044
66.0000	1.0000	1.0000	1.0014	0.1398	0.1098	94.5083
67.0000	1.0000	1.0000	1.0438	4.3845	1.5207	23.9665
68.0000	1.0000	1.0000	0.9642	3.5759	0.0734	96.3287
69.0000	1.0000	1.0000	1.0605	6.0481	0.1509	92.4525
70.0000	1.0000	1.0000	0.9898	1.0219	0.0596	97.0217
71.0000	1.0000	1.0000	0.9624	3.7559	0.6760	66.2015
72.0000	1.0000	1.0000	1.0082	0.8167	0.3232	83.8379
73.0000	1.0000	1.0000	0.9698	3.0219	0.1123	94.3832
74.0000	1.0000	2.0000	0.9741	2.5883	2.0374	1.8722
75.0000	1.0000	1.0000	0.9759	2.4105	1.0727	46.3674
76.0000	1.0000	1.0000	0.9827	1.7292	0.6210	68.9515
77.0000	1.0000	1.0000	1.0521	5.2091	1.0619	46.9035
78.0000	1.0000	1.0000	0.9811	1.8875	1.8186	9.0699
79.0000	1.0000	1.0000	1.0275	2.7518	1.8777	6.1142
80.0000	1.0000	1.0000	1.0125	1.2487	1.8690	6.5496
81.0000	1.0000	1.0000	0.9860	1.4039	0.2355	88.2274
82.0000	1.0000	2.0000	0.9454	5.4598	1.9270	3.6498
83.0000	1.0000	2.0000	1.0426	4.2552	1.9568	2.1591
84.0000	1.0000	1.0000	0.9331	6.6905	0.3172	84.1391
85.0000	1.0000	2.0000	1.0295	2.9512	2.0230	1.1488
86.0000	1.0000	1.0000	0.9978	0.2215	1.1471	42.6435
87.0000	1.0000	1.0000	0.9673	3.2689	1.3060	34.6999
88.0000	1.0000	1.0000	0.9824	1.7612	1.6868	15.6576
89.0000	1.0000	1.0000	0.9858	1.4203	1.8035	9.8233
90.0000	1.0000	2.0000	0.9491	5.0859	1.9338	3.3110
91.0000	1.0000	1.0000	1.0304	3.0429	1.1066	44.6690
92.0000	1.0000	1.0000	0.9860	1.3980	1.2714	36.4295
93.0000	1.0000	1.0000	1.0152	1.5193	1.8161	9.1952
94.0000	1.0000	1.0000	0.9829	1.7131	0.9968	50.1594
95.0000	1.0000	1.0000	1.0166	1.6630	1.7364	13.1792
96.0000	1.0000	1.0000	1.0085	0.8489	1.8879	5.6073
97.0000	1.0000	1.0000	1.0017	0.1725	1.8884	5.5808
98.0000	1.0000	1.0000	0.9258	7.4243	1.0560	47.1997
99.0000	1.0000	1.0000	0.9930	0.6984	1.2081	39.5954
100.0000	1.0000	1.0000	1.1171	11.7059	1.6573	17.1355

Table 3.2 (cont.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
101.0000	2.0000	2.0000	0.0484	95.1601	1.9728	1.3613
102.0000	2.0000	2.0000	0.1558	84.4222	2.0232	1.1586
103.0000	2.0000	1.0000	1.0020	0.2001	2.0469	2.3439
104.0000	2.0000	2.0000	0.9540	4.6045	2.0260	1.3009
105.0000	2.0000	2.0000	1.0455	4.5522	2.0077	0.3861
106.0000	2.0000	2.0000	0.8110	18.9005	2.0348	1.7392
107.0000	2.0000	2.0000	0.5297	47.0302	2.0331	1.6560
108.0000	2.0000	2.0000	0.5364	46.3611	1.9782	1.0922
109.0000	2.0000	2.0000	0.0083	99.1708	2.0275	1.3739
110.0000	2.0000	2.0000	0.4728	52.7199	2.0554	2.7697
111.0000	2.0000	2.0000	0.0011	99.8885	2.0224	1.1190
112.0000	2.0000	2.0000	0.0185	98.1471	1.9888	0.5619
113.0000	2.0000	2.0000	0.9030	9.6996	2.0078	0.3906
114.0000	2.0000	2.0000	0.0968	90.3173	2.0159	0.7950
115.0000	2.0000	2.0000	0.0959	90.4091	2.0099	0.4928
116.0000	2.0000	2.0000	0.2978	70.2246	1.9651	1.7439
117.0000	2.0000	2.0000	0.4247	57.5277	2.0167	0.8340
118.0000	2.0000	2.0000	0.0035	99.6485	2.0015	0.0761
119.0000	2.0000	2.0000	0.0032	99.6797	1.9860	0.6999
120.0000	2.0000	2.0000	0.0339	96.6089	2.0070	0.3516
121.0000	2.0000	2.0000	0.0022	99.7844	2.0079	0.3958
122.0000	2.0000	2.0000	0.5709	42.9095	1.9776	1.1215
123.0000	2.0000	2.0000	0.0675	93.2529	1.9989	0.0564
124.0000	2.0000	2.0000	0.4317	56.8282	1.9739	1.3063
125.0000	2.0000	2.0000	0.3318	66.8162	2.0354	1.7691
126.0000	2.0000	2.0000	0.0009	99.9147	2.0176	0.8790
127.0000	2.0000	2.0000	0.3663	63.3676	1.9928	0.3585
128.0000	2.0000	2.0000	0.7197	28.0292	1.9717	1.4147
129.0000	2.0000	2.0000	0.0390	96.1042	1.9995	0.0238
130.0000	2.0000	2.0000	0.0350	96.4977	1.9482	2.5913
131.0000	2.0000	2.0000	0.0037	99.6336	2.0423	2.1163
132.0000	2.0000	2.0000	0.1828	81.7202	2.0069	0.3434
133.0000	2.0000	2.0000	0.0036	99.6424	2.0320	1.6005
134.0000	2.0000	2.0000	0.1841	81.5933	2.0081	0.4038
135.0000	2.0000	2.0000	0.0724	92.7631	1.9854	0.7280
136.0000	2.0000	2.0000	0.1613	83.8725	2.0228	1.1377
137.0000	2.0000	2.0000	0.0328	96.7163	1.9816	0.9211
138.0000	2.0000	2.0000	0.0386	96.1431	1.9735	1.3272
139.0000	2.0000	2.0000	0.0739	92.6112	2.0160	0.7980
140.0000	2.0000	2.0000	0.0483	95.1715	1.9943	0.2843
141.0000	2.0000	2.0000	0.2668	73.3204	1.9887	0.5655
142.0000	2.0000	2.0000	0.0004	99.9595	2.0041	0.2031
143.0000	2.0000	2.0000	0.0492	95.0764	2.0055	0.2762
144.0000	2.0000	2.0000	0.1101	88.9882	2.0222	1.1094
145.0000	2.0000	1.0000	1.0044	0.4396	2.0095	0.4735
146.0000	2.0000	2.0000	0.0041	99.5927	2.0358	1.7883
147.0000	2.0000	2.0000	0.2805	71.9527	1.9818	0.9096
148.0000	2.0000	2.0000	0.2607	73.9256	2.0305	1.5233
149.0000	2.0000	2.0000	0.0980	90.1971	1.9842	0.7876
150.0000	2.0000	2.0000	0.4568	54.3194	1.9808	0.9590

Table 3.2 (cont.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
151.0000	2.0000	2.0000	0.0630	93.7048	2.0144	0.7198
152.0000	2.0000	2.0000	0.0482	95.1759	2.0039	0.1942
153.0000	2.0000	2.0000	1.0019	0.1945	2.0030	0.1502
154.0000	2.0000	2.0000	0.0258	97.4157	1.9612	1.9398
155.0000	2.0000	2.0000	0.3745	62.5493	2.0463	2.3127
156.0000	2.0000	2.0000	0.2262	77.3792	2.0506	2.5283
157.0000	2.0000	2.0000	0.0003	99.9720	1.9834	0.8321
158.0000	2.0000	2.0000	0.0216	97.8413	1.9750	1.2512
159.0000	2.0000	2.0000	0.0039	99.6124	1.9861	0.6941
160.0000	2.0000	1.0000	1.0085	0.8477	2.0464	2.3194
161.0000	2.0000	2.0000	0.5277	47.2270	1.9570	2.1475
162.0000	2.0000	2.0000	0.4875	51.2548	2.0240	1.2016
163.0000	2.0000	2.0000	0.6499	35.0126	1.9942	0.2889
164.0000	2.0000	1.0000	0.9811	1.8901	1.9458	2.7113
165.0000	2.0000	2.0000	0.0875	91.2489	2.0075	0.3766
166.0000	2.0000	2.0000	0.0599	94.0138	2.0058	0.2910
167.0000	2.0000	2.0000	0.0240	97.6000	1.9842	0.7901
168.0000	2.0000	2.0000	0.4628	53.7166	1.9922	0.3881
169.0000	2.0000	2.0000	0.0003	99.9658	1.9965	0.1741
170.0000	2.0000	2.0000	0.5690	43.1032	2.0186	0.9323
171.0000	2.0000	2.0000	0.3324	66.7573	1.9688	1.5593
172.0000	2.0000	2.0000	0.2539	74.6121	1.9914	0.4316
173.0000	2.0000	2.0000	0.1354	86.4613	2.0251	1.2537
174.0000	2.0000	2.0000	0.0101	98.9862	2.0030	0.1507
175.0000	2.0000	2.0000	0.0110	98.8977	2.0114	0.5721
176.0000	2.0000	2.0000	0.0001	99.9873	1.9933	0.3326
177.0000	2.0000	2.0000	0.1848	81.5162	1.9531	2.3460
178.0000	2.0000	2.0000	0.4435	55.6484	2.0249	1.2442
179.0000	2.0000	2.0000	0.0002	99.9835	1.9969	0.1530
180.0000	2.0000	2.0000	0.0052	99.4797	2.0114	0.5701
181.0000	2.0000	2.0000	0.9698	3.0186	2.0018	0.0887
182.0000	2.0000	2.0000	0.0086	99.1419	2.0097	0.4832
183.0000	2.0000	2.0000	0.9370	6.3003	2.0009	0.0470
184.0000	2.0000	2.0000	0.0002	99.9801	1.9628	1.8592
185.0000	2.0000	2.0000	0.1920	80.8027	2.0088	0.4409
186.0000	2.0000	2.0000	0.4221	57.7878	2.0205	1.0265
187.0000	2.0000	2.0000	0.5677	43.2302	2.0109	0.5464
188.0000	2.0000	2.0000	0.1071	89.2944	1.9880	0.6016
189.0000	2.0000	2.0000	0.1245	87.5472	1.9981	0.0944
190.0000	2.0000	2.0000	0.4564	54.3586	2.0151	0.7575
191.0000	2.0000	2.0000	0.0004	99.9618	1.9745	1.2749
192.0000	2.0000	2.0000	0.5734	42.6560	1.9813	0.9348
193.0000	2.0000	2.0000	0.5854	41.4564	2.0245	1.2270
194.0000	2.0000	2.0000	0.0294	97.0592	1.9709	1.4528
195.0000	2.0000	2.0000	0.6830	31.7014	1.9739	1.3058
196.0000	2.0000	2.0000	0.0033	99.6726	2.0494	2.4704
197.0000	2.0000	2.0000	0.0019	99.8144	2.0085	0.4260
198.0000	2.0000	2.0000	0.0849	91.5147	2.0205	1.0225
199.0000	2.0000	2.0000	0.0241	97.5858	1.9666	1.6698
200.0000	2.0000	2.0000	0.0008	99.9228	2.0126	0.6290

Chapter 4

Clustering

Clustering represents one of the broader and most sought after data analysis techniques. The vast appeal of clustering techniques has to do with the fact that realistic data structures are often the aggregate of a disjointed set of data groups,, as so characterized by common consensus in visual observations, at least for low dimensionality feature vectors where such visual appraisals can be directly executed. There are numerous algorithms and a voluminous literature on the topic of cluster analysis. One distinguishes hard, probabilistic and fuzzy clustering approaches. The hard techniques assign a data point to one and only one cluster. The fuzzy techniques have assumed more prominence in the last few years because they assign a data point to all clusters with the assignment to a given cluster being characterize by a degree of membership with a value that varies between 0 and 1. Thus, if a data point is very far away from a cluster center the membership value may be close to 0 while if a data point is very near to a cluster center its degree of membership is close to 1. This is a much more intuitively appealing quantitative environment to imbed the clustering problem into than the binary choice of the hard clustering techniques.

Clustering can become a classification technique all by itself. However, for our purposes clustering is to act as a preprocessing method that allows identification of compact groups of data that Gaussian Potential Function Units can be defined for. Thus, clustering represents a bandwidth compression technique for us. The clustering algorithm we chose is the fuzzy

c-means algorithm developed by Dunn [17] and extended by Bezdek [3]. It is the most prominent fuzzy clustering algorithm with significant applications in the biomedical area [1].

4.1 Fuzzy c-means Algorithm

Discrimination of data sets for realistic problems is a difficult task because the probabilistic distributional data generating mechanisms and the ensuing feature space geometric configurations are, typically not known, a priori. Clustering algorithms are thus useful in allowing the partitioning of the data into a set of geometrically compact elements which can, in turn, be encoded with the Gaussian Potential Functions. The typical clustering algorithms assign an element in the data set to one and only one cluster. A fuzzy clustering technique enhances flexibility by assigning membership function values to each element of all clusters.

To show how this is accomplished, let the data set be denoted by $X = \{x_1, ...x_n\}$ where each element is called a feature vector, i.e., $x_k = [x_{k1}, ..., x_{kq}]$ with x_{kj} being the jth feature of the kth sample in the data set. The clustering criterion is to have the elements of a cluster be as similar (in a distance metric sense) as possible while elements of different clusters should be as dissimilar as possible. The Euclidean distance between two elements ($d(x_k, x_j) = \| x_k - x_j \|^2$) is a common and good distance metric.

Each cluster of the data set X can be mapped into fuzzy subsets $S_i, i = 1, ...c$ by a membership function $\mu_{S_i} : X \rightarrow [0, 1]$. In other words, for a feature vector x_k, its degree of belonging to cluster i is given by μ_{ik}, the membership of x_k to the subset S_i, i.e, $\mu_{ik} = \mu_{Si}(x_k)$. Let V_{cn} be the set of all real cxn matrices with $2 \leq c < n$. The matrix $U = [\mu_{ik}] \in V_{cn}$ is called a fuzzy c-partition matrix if it satisfies the following conditions:

$$\mu_{ik} \in [0, 1], \quad 1 \leq i \leq c, \quad 1 \leq k \leq n \tag{4.1}$$

$$\sum_{i=1}^{c} \mu_{ik} = 1, \quad 1 \leq k \leq n \tag{4.2}$$

$$0 < \sum_{k=1}^{n} \mu_{ik} < n, \quad 1 \leq i \leq c \tag{4.3}$$

The last two conditions imply that the "total membership" of an element is normalized to 1 and that it can not belong to more clusters than there exist. The location of a cluster is represented by its "cluster center" or its prototype $v_i = [v_{i1}, ..., v_{iq}], \quad i = 1, ..., c$. The v_i, in general, may not correspond to any element of X.

The basic fuzzy-c means problem now is to minimize the following objective function for m>1:

$$min\ J_m(U; v) = \sum_{k=1}^{n} \sum_{i=1}^{c} (\mu_{ik})^m \| x_k - v_i \|^2 \tag{4.4}$$

such that $U \in V_{cn}$ and $v \in R^{c \times n}$. Differentiating the objective function with respect to v_i (for fixed U) and with respect to μ_{ik} (for fixed v), and applying the condition $\sum_{i=1}^{c} \mu_{ik} = 1$, yields

$$v_i = \frac{1}{\sum_{k=1}^{n} (\mu_{ik})^m} \sum_{k=1}^{n} (\mu_{ik})^m x_k, \quad i = 1, ...c \tag{4.5}$$

$$\mu_{ik} = \frac{\| x_k - v_i \|^{\frac{-2}{m-1}}}{\sum_{j=1}^{c} \| x_k - v_j \|^{\frac{-2}{m-1}}}, \quad i = 1, ..c; \quad k = 1, ..., n \tag{4.6}$$

The parameter m is an exponential weight, used to reduce the influence of relatively distant points. That is, the influence of small μ_{ik} (points further away from v_i) is penalized

compared to that of large μ_{ik} (points close to v_i). The iteration algorithm (Bezdek [3]) to solve the optimal fuzzy c-means cluster problem comprises the following steps:

Step 1. Choose c ($2 \leq c \leq n$) and m ($1 < m < \infty$). Initialize $U^{(0)}$ and set l = 0.

Step 2. Calculate the c fuzzy cluster centers v_i^l from 4.5 by using U^l.

Step 3. Calculate the new membership function matrix U^{l+1} through 4.6, by using $v_i^{(l)}$ if $x_k \neq v_i^{(l)}$; else set $\mu_{jk} = 1$ for j=i or $\mu_{jk} = 0$ for j $\neq$ i.

Step 4. Calculate $\Delta = \| U^{(l+1)} - U^{(l)} \|$. If $\Delta > \epsilon$ set $l = l + 1$ and go to Step 2; otherwise, stop.

The fuzzy c-means algorithm assumes that the number,c, of clusters is a priori known (below we show how such a practically unrealistic assumption can be circumvented). Given the c cluster centers v_i the degree of membership of data point x_k to cluster i is:

$$\mu_{ik} = \frac{\| x_k - v_i \|^{\frac{-2}{m-1}}}{\sum_{j=1}^{c} \| x_k - v_j \|^{\frac{-2}{m-1}}}, \quad i = 1, ..c; \quad k = 1, ..., n \tag{4.7}$$

To evaluate the efficacy of such a definition let us first set the value of m to 2. Then, the above expression becomes:

$$\mu_{ik} = \frac{\| x_k - v_i \|^{-2}}{\sum_{j=1}^{c} \| x_k - v_j \|^{-2}}, \quad i = 1, ..c; \quad k = 1, ..., n \tag{4.8}$$

The degree of membership of the data point x_k to the cluster i, μ_{ik}, is the ratio of the inverse square distance of x_k from the cluster center v_i to the sum of the inverse square distances of the same data point from the c clusters. If the data point x_k is close to the center of cluster i and far from the remaining cluster centers then the membership value will be close to 1, an intuitively satisfying result. If the data point x_k is far away from the center of cluster i and close to some other center, j say, then the numerator of 4.8 will be small and the denominator large yielding a membership value close to 0, an equally intuitively

appealing circumstance. We thus see that the definition of the degree of membership of a data point to a given cluster, as given by 4.8 is in harmony with an acceptable geometric interpretation of the clustering process. It is now noted that higher values of m imply more severe weighting for data points further away from the cluster centers.

The cluster centers are defined through the expression:

$$v_i = \frac{1}{\sum_{k=1}^{n}(\mu_{ik})^m}\sum_{k=1}^{n}(\mu_{ik})^m x_k, \quad i = 1, ...c \tag{4.9}$$

Thus, the cluster center i is nothing more than the mean of the data points weighted by their degree of membership to the cluster i.

The fuzzy c-means algorithm iterates through the above expressions for cluster membership values and cluster centers as a process that has been shown to minimize the objective function

$$J_m(U; v) = \sum_{k=1}^{n}\sum_{i=1}^{c}(\mu_{ik})^m \parallel x_k - v_i \parallel^2 \tag{4.10}$$

which represents the fundamental fuzzy c-means algorithmic aim which is to minimize the sum of the weighted square distances of the data points from the cluster centers. The iterative algorithm converges to at least a local minimum.

4.2 Example

To demonstrate the fuzzy c-means clustering algorithm an arbitrary set of four two-dimensional clusters was generated, as shown in Figure 4.1. Each cluster consists of thirty feature vectors generated by randomly perturbing through a uniform distribution the nominal center values of each cluster which was arbitrarily selected as follows:

$$
\begin{aligned}
&Cluster\ 1\ Center: && (\ 7,\ 7) \\
&Cluster\ 2\ Center: && (22,\ 7) \\
&Cluster\ 3\ Center: && (14, 22) \\
&Cluster\ 4\ Center: && (22, 22)
\end{aligned}
\tag{4.11}
$$

The so generated feature vectors for each cluster are shown in Figures 4.2, 4.3, 4.4 and 4.5. The feature vectors are plotted as waveforms for illustration purposes. The elements of the feature vectors are the ordinate values corresponding to the integer abscissa values 1 and 2.

The membership matrix U has four rows (corresponding to the four clusters) and one hundred and twenty columns (corresponding to the number of data points or feature vectors). It is arbitrarily initialized with the values:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & \dots \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & \dots \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & \dots
\end{bmatrix}
\tag{4.12}
$$

The fuzzy c-means algorithm was iterated 11 times until the stopping error criterion became less than 0.000001. The error iteration history is shown in Figure 4.6. The four cluster centers as determined by the fuzzy c-means algorithm are shown below (and in comparison to the designed centers):

$$
\begin{aligned}
&Fuzzy\ c-means\ Cluster\ 1\ Center: && (\ 6.9392,\ 7.1862)\ versus && (\ 7,\ 7) \\
&Fuzzy\ c-means\ Cluster\ 2\ Center: && (21.9267,\ 6.7180)\ versus && (22,\ 7) \\
&Fuzzy\ c-means\ Cluster\ 3\ Center: && (14.2462, 21.4249)\ versus && (14, 22) \\
&Fuzzy\ c-means\ Cluster\ 4\ Center: && (22.5265, 22.2849)\ versus && (22, 22)
\end{aligned}
\tag{4.13}
$$

The fuzzy c-means centers are shown in Figure 4.7 superimposed onto the four clusters. The membership matrix U is shown in Table 4.1. There are four rows corresponding to the number of clusters and one hundred and twenty columns corresponding to the number of total feature vectors. The algorithm assigns a given element to the cluster that it exhibits the highest membership value for. Thus, Table 4.2 shows the individual cluster assignment for each data point. Column (1) identifies the feature vector, column (2) is the originally designed cluster assignment, column (3) is the algorithm derived cluster assignment and columns (4), (5), (6) and (7) are the membership values for each cluster established by the algorithm. It is noted that the designed cluster assignment number has no relation to the algorithm derived cluster number. In other words, the originally designated cluster 1 may be called cluster 2 or 3 or 4 by the algorithm. The basic focus of the fuzzy c-means algorithm solution are the data points assigned to each cluster.

Figures 4.8 through 4.19 show the membership values of the data points, with 10 data points plotted per Figure. Thus, Figures 4.8, 4.9 and 4.10 show that the first thirty data points exhibit the highest memberhip values for cluster no. 2. Figures 4.11, 4.12 and 4.13 show that the next thirty data points have the highest membership values for cluster no. 1. Figures 4.14, 4.15 and 4.16 show the association of the next thirty points with cluster no. 4 with respect to which they have the highest membership values. Finally, Figures 4.17, 4.18 and 4.19 clearly show the assignment of the last thirty points to the cluster no. 3.

4.3 Selection of c

The example above assumed that the number of clusters is already known. In practice one is not expected to often know, a priori, the expected number of clusters the data can be partitioned into. Xie and Beni [16] proposed a measure whose minimization aims at identifying the "right" number of clusters present in the data. This cluster validity measure is defined as:

$$S = \frac{\sum_{i=1}^{c}\sum_{k=1}^{n}\mu_{ik}^{m} \| x_k - v_i \|^2}{n * min \| v_i - v_k \|^2} \tag{4.14}$$

and can be given the following interpretation. First we note that the term ${\mu_{ik}}^m \| x_k - v_i \|^2$ represents the fuzzy square distance or square deviation of data point k from the cluster center i. For each cluster i, the sum of the squares of the fuzzy deviations is called the variation of cluster i. Thus, the expression $(\sum_{i=1}^{c}\sum_{k=1}^{n}\mu_{ik}^{m} \| x_k - v_i \|^2)/n$ represents the average variation of the data points, called the *compactness* of the data. This average variation (which has an interpretation analogous to the statistical variance) is then referenced to the smallest distance among the cluster centers to yield the validity measure 4.14. The validitity measure 4.14 is therefore stuctured to have a smaller value for a configuration of clusters versus data that is more "compact" relative to the cluster centers separation, an intuitively appealing formulation.

The above described validity measure was tested by noting its values as c (the number of assumed clusters) of the fuzzy c-means algorithm was varied from 2 to 10. The results are shown below and plotted in Figure 4.20:

$$\begin{aligned}
When\ No.\ of\ Clusters &= 2\ S = 0.1573 \\
When\ No.\ of\ Clusters &= 3\ S = 0.2960 \\
When\ No.\ of\ Clusters &= 4\ S = 0.0610 \\
When\ No.\ of\ Clusters &= 5\ S = 0.4114 \\
When\ No.\ of\ Clusters &= 6\ S = 1.2170 \\
When\ No.\ of\ Clusters &= 7\ S = 1.3417 \\
When\ No.\ of\ Clusters &= 8\ S = 1.9601 \\
When\ No.\ of\ Clusters &= 9\ S = 1.9081 \\
When\ No.\ of\ Clusters &= 10\ S = 1.5854
\end{aligned} \tag{4.15}$$

It is noted that a minimum occurs when the selected number of clusters is 4, matching the designed actual cluster number.

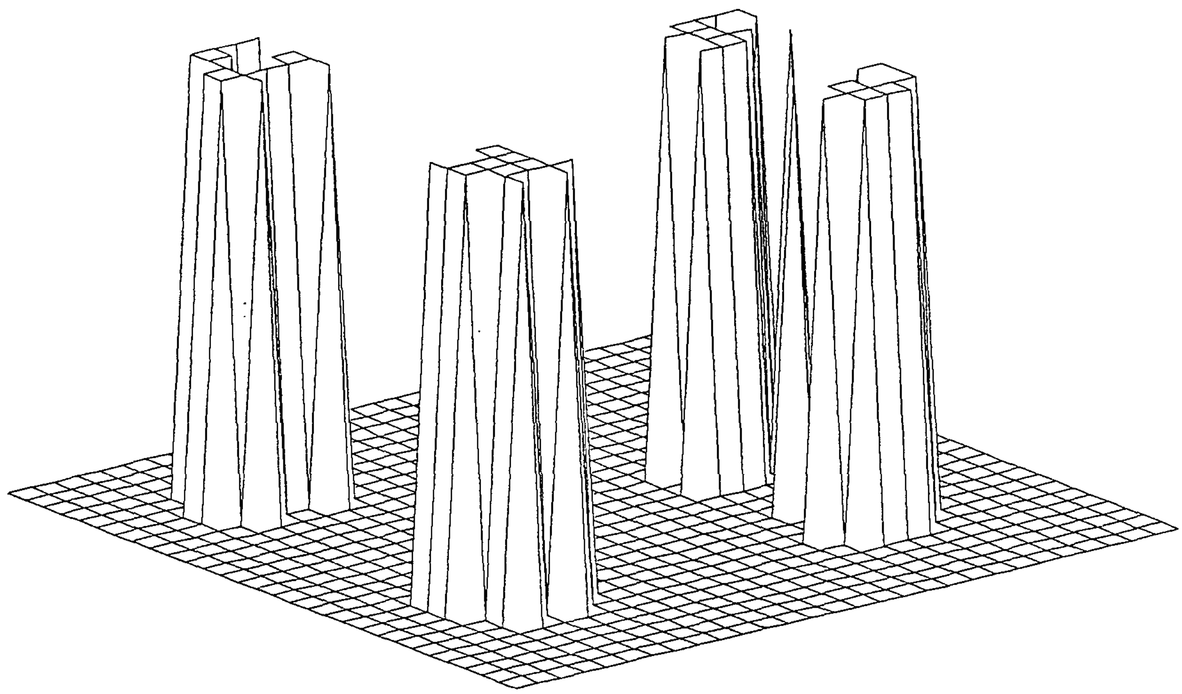

Figure 4.1: Four clusters distribution.

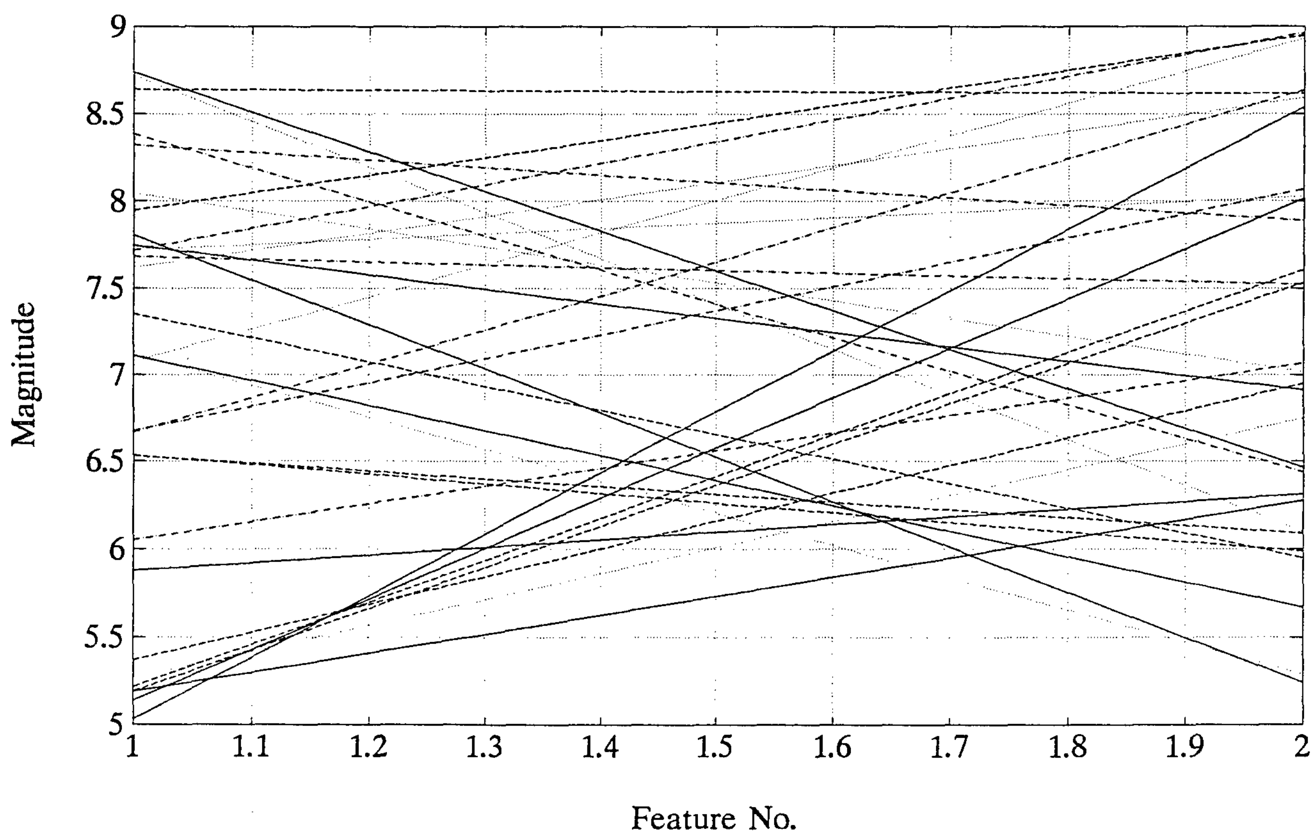

Figure 4.2: Cluster 1 feature vectors plotted as waveforms.

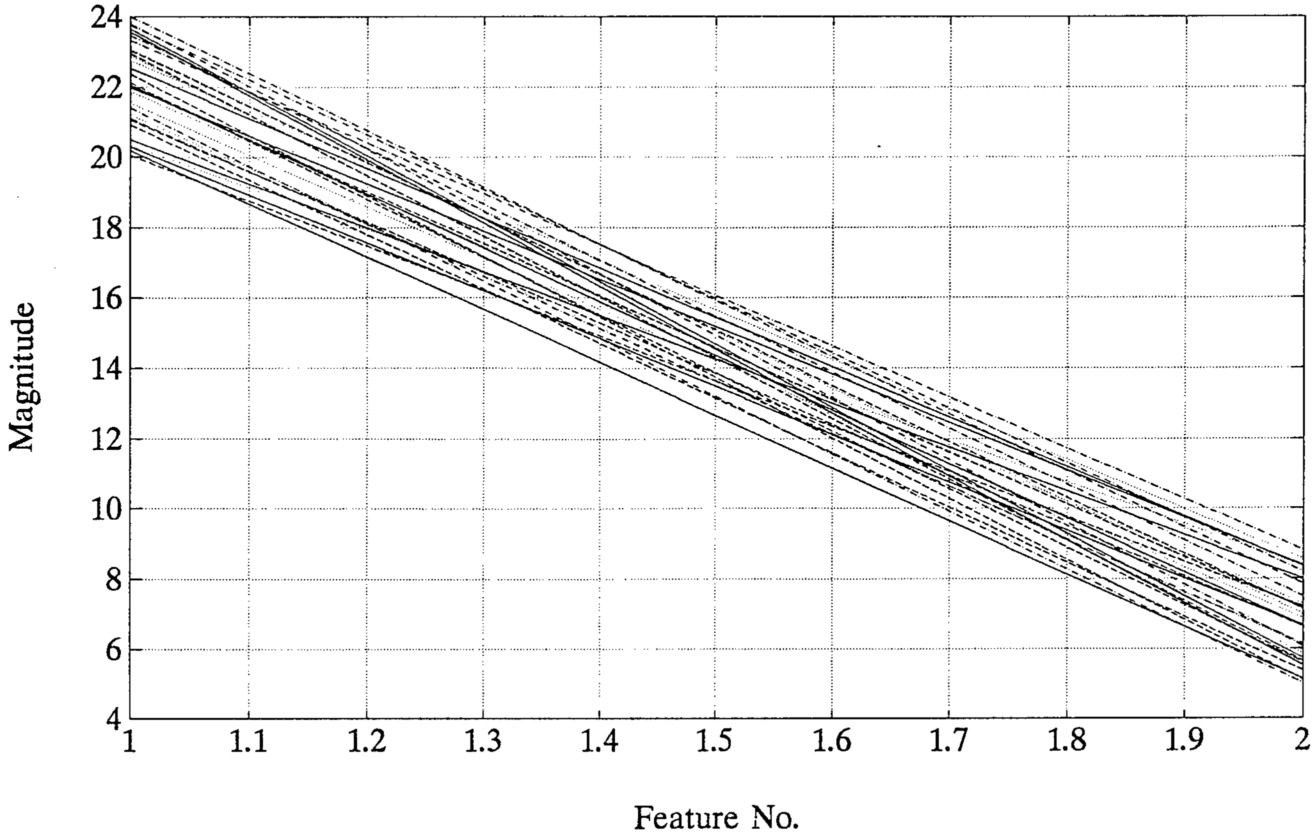

Figure 4.3: Cluster 2 feature vectors plotted as waveforms.

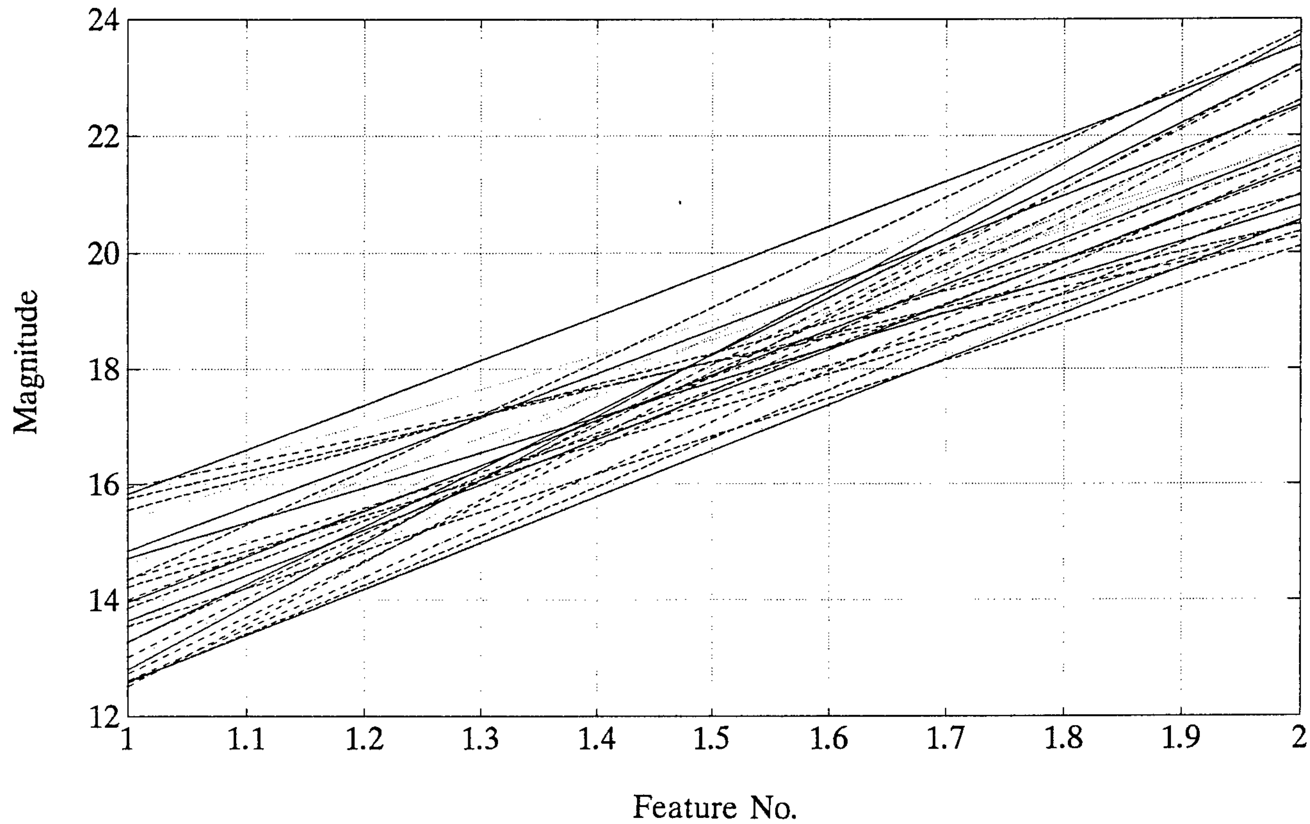

Figure 4.4: Cluster 3 feature vectors plotted as waveforms.

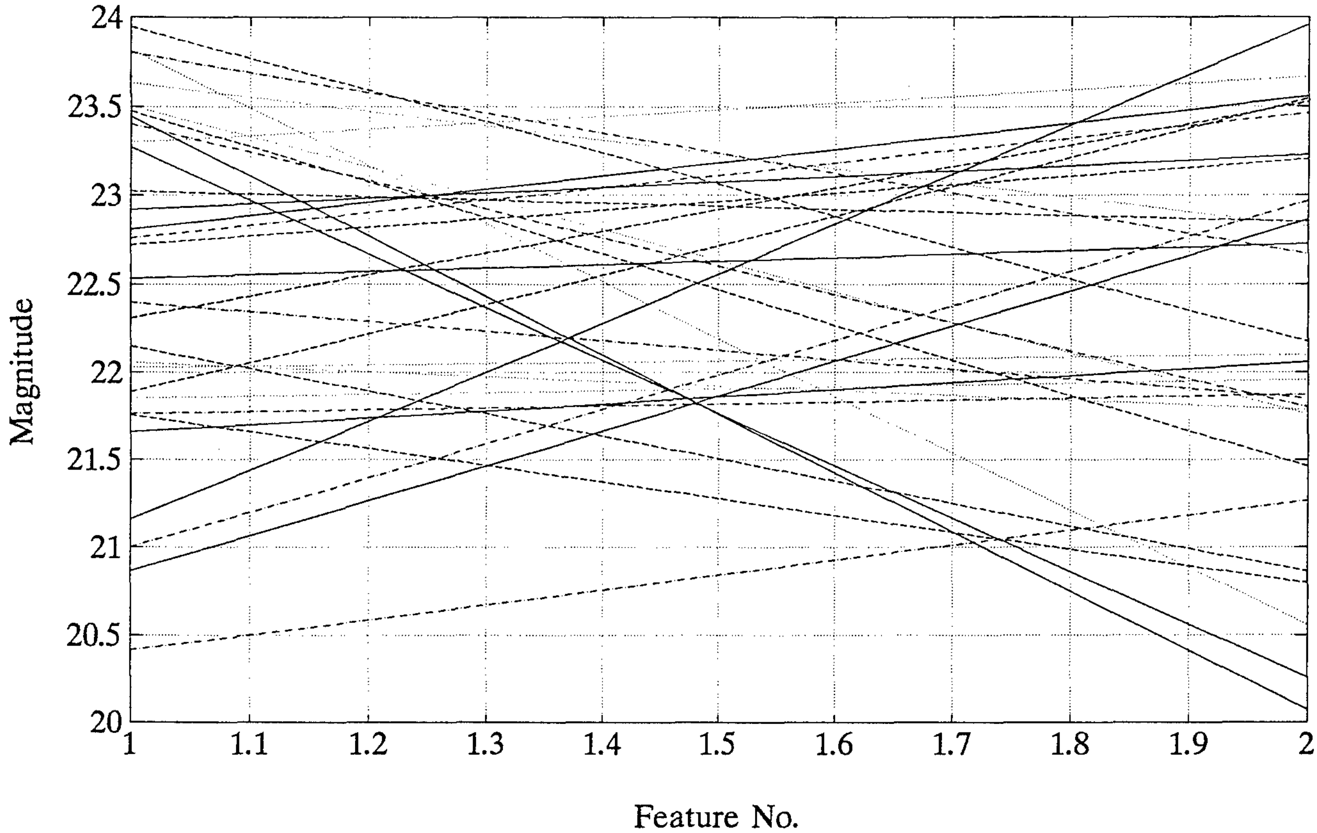

Figure 4.5: Cluster 4 feature vectors plotted as waveforms.

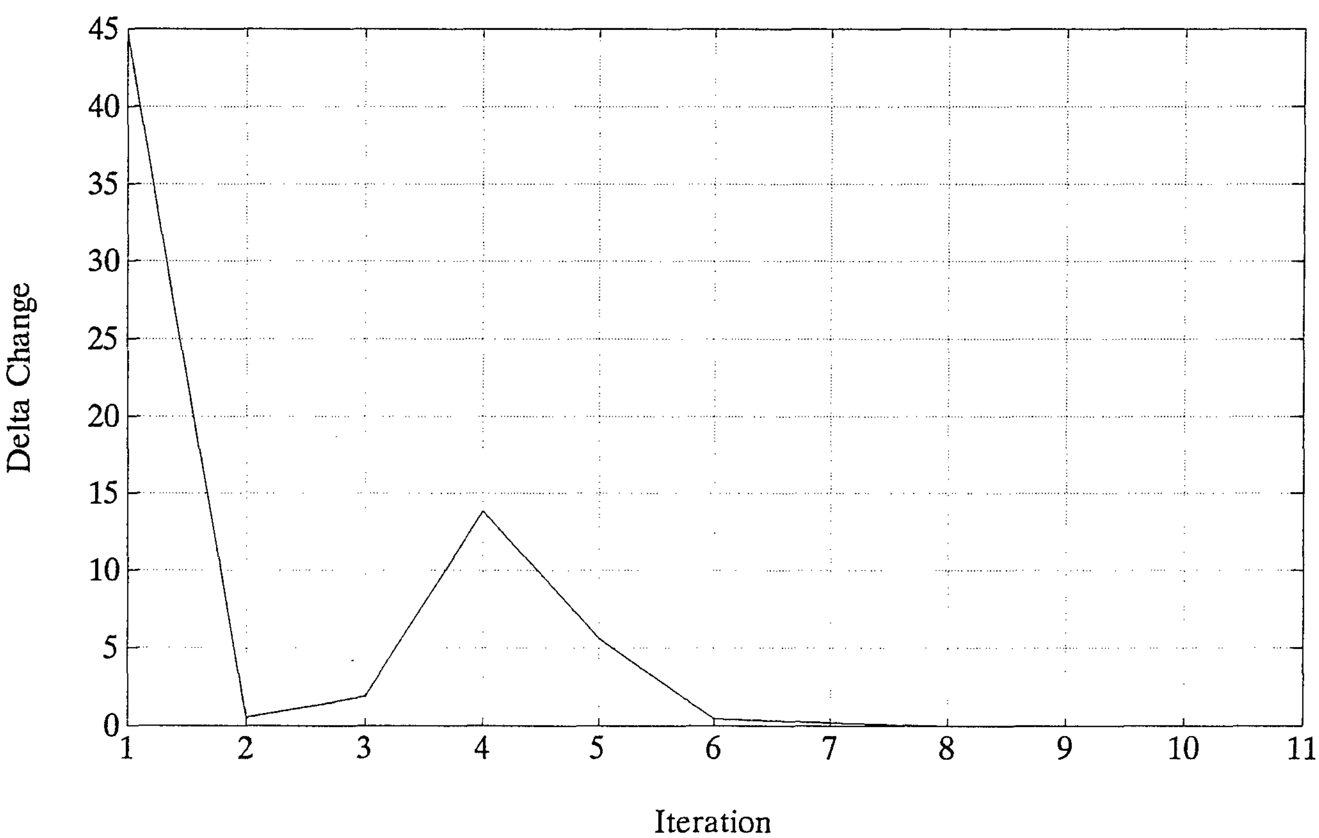

Figure 4.6: Delta membership function change iteration history.

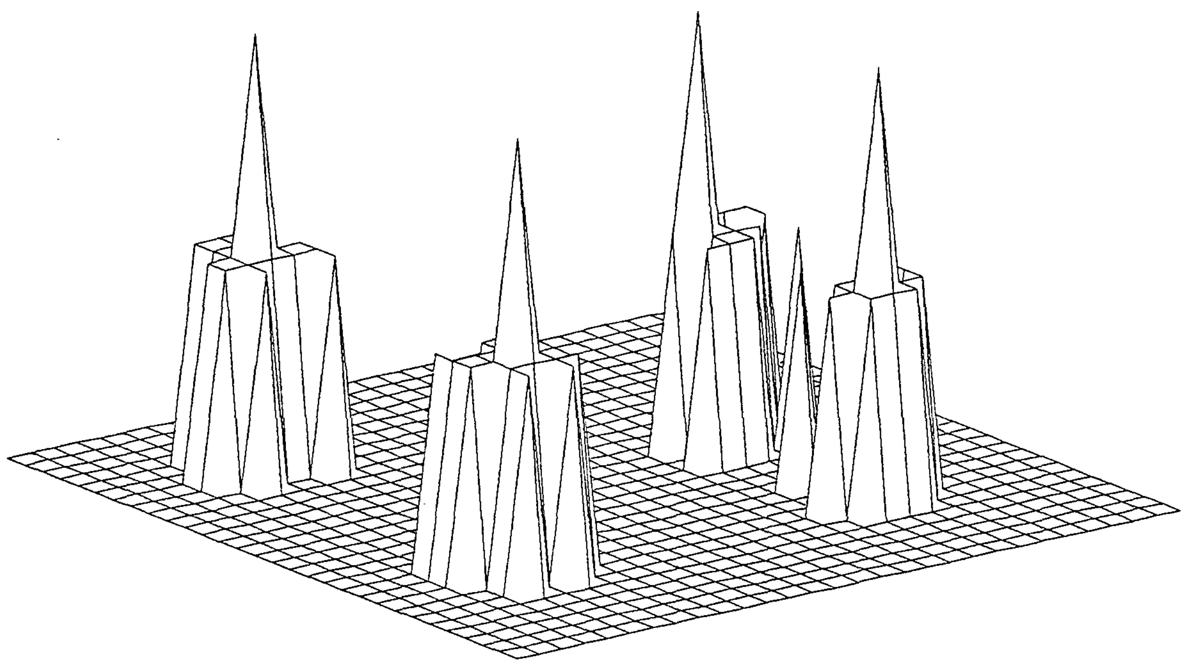

Figure 4.7: Fuzzy c-means algorithm derived clusters and cluster centers.

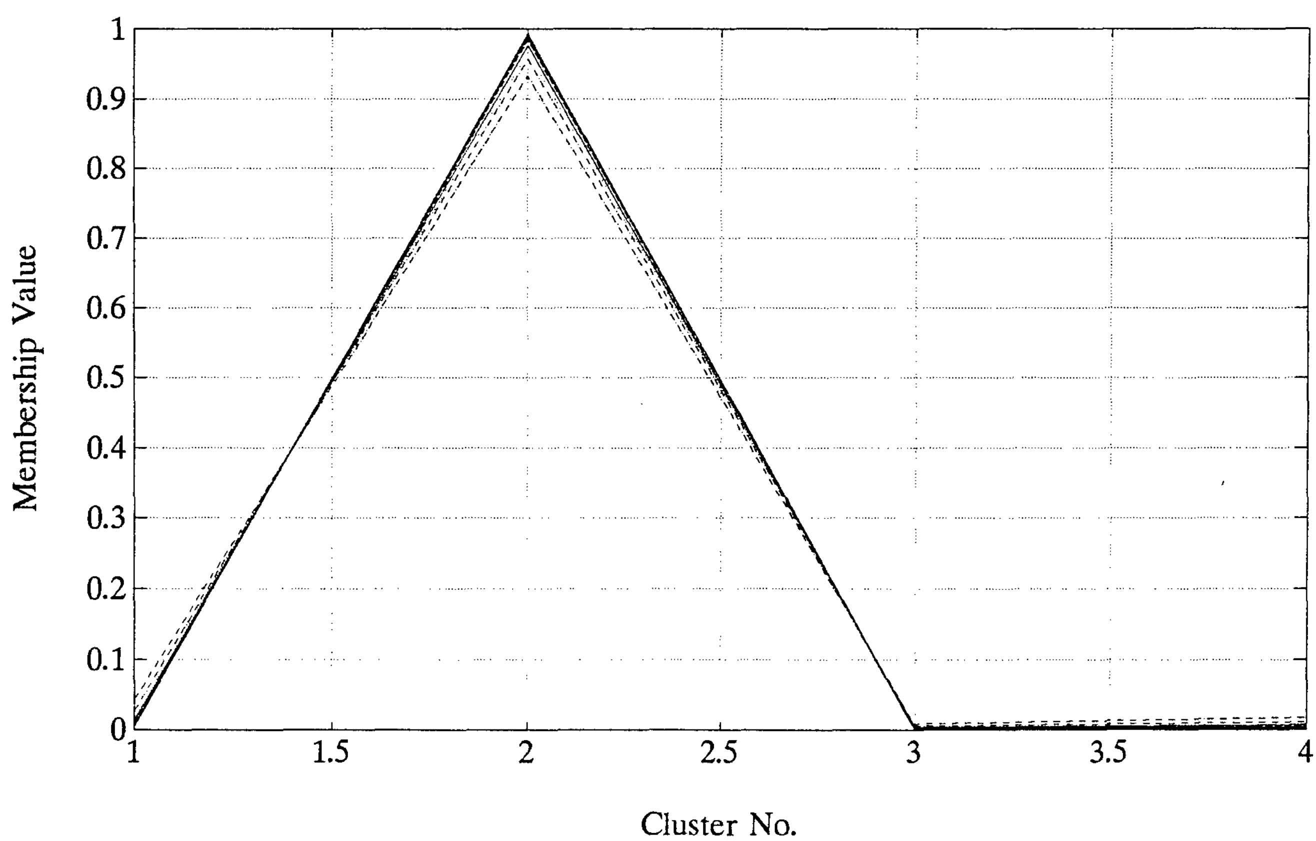

Figure 4.8: Membership values for feature vectors 1-10.

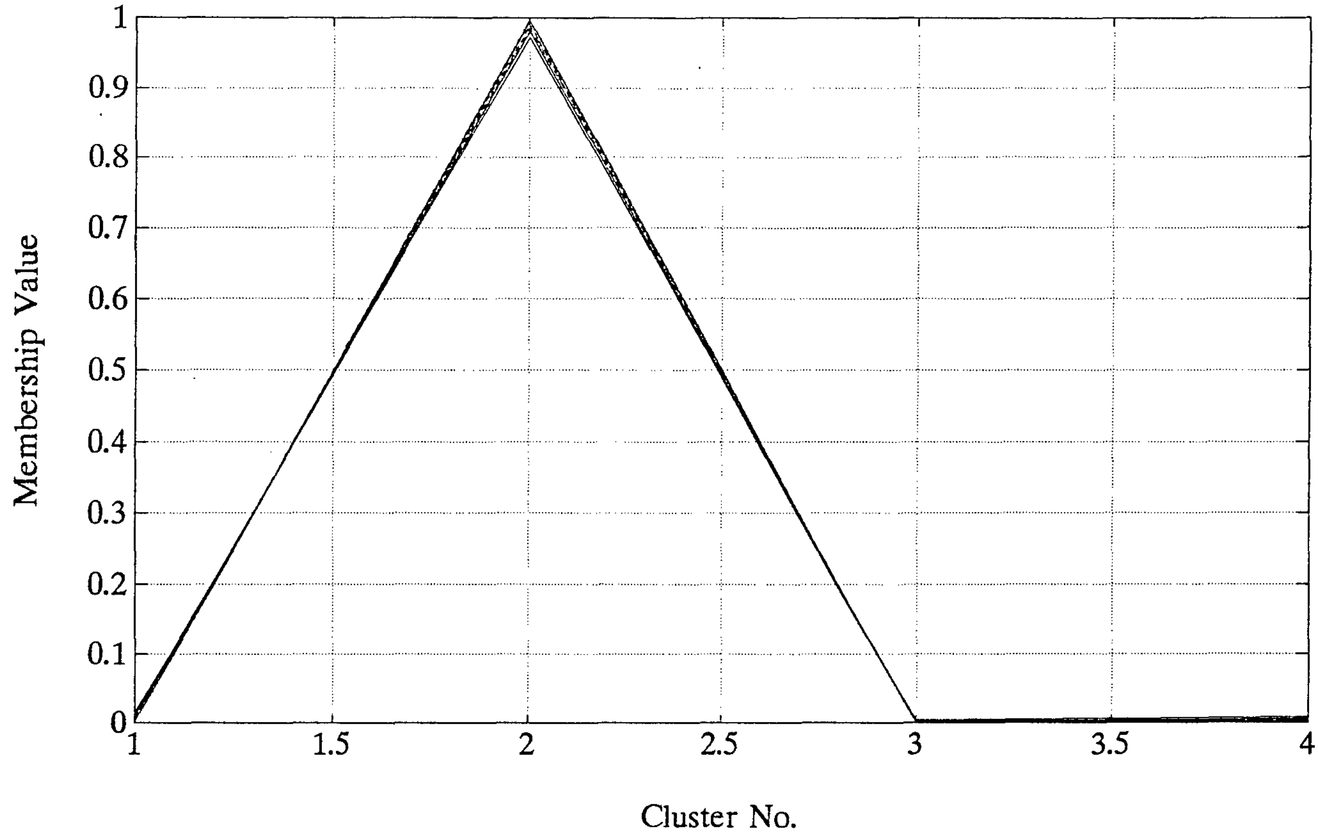

Figure 4.9: Membership values for feature vectors 11-20.

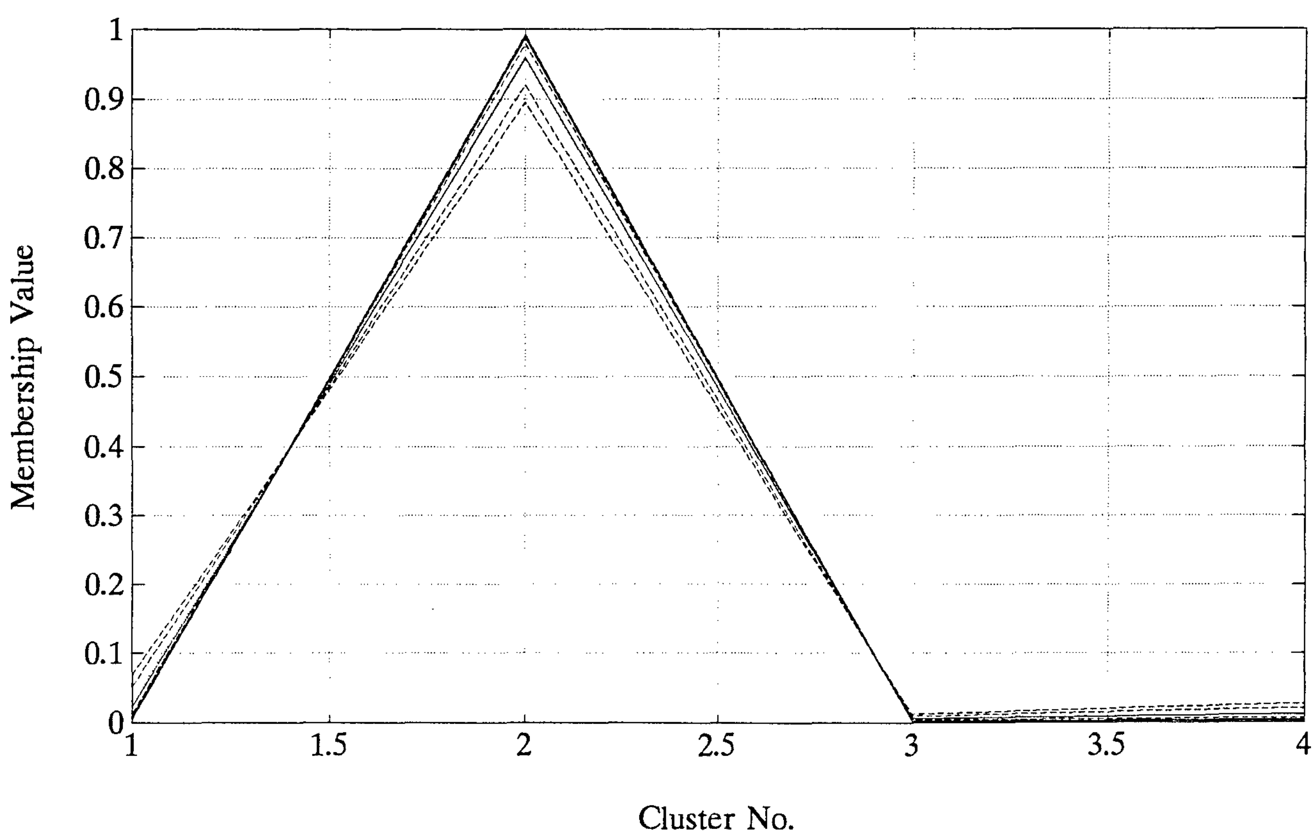

Figure 4.10: Membership values for feature vectors 21-30.

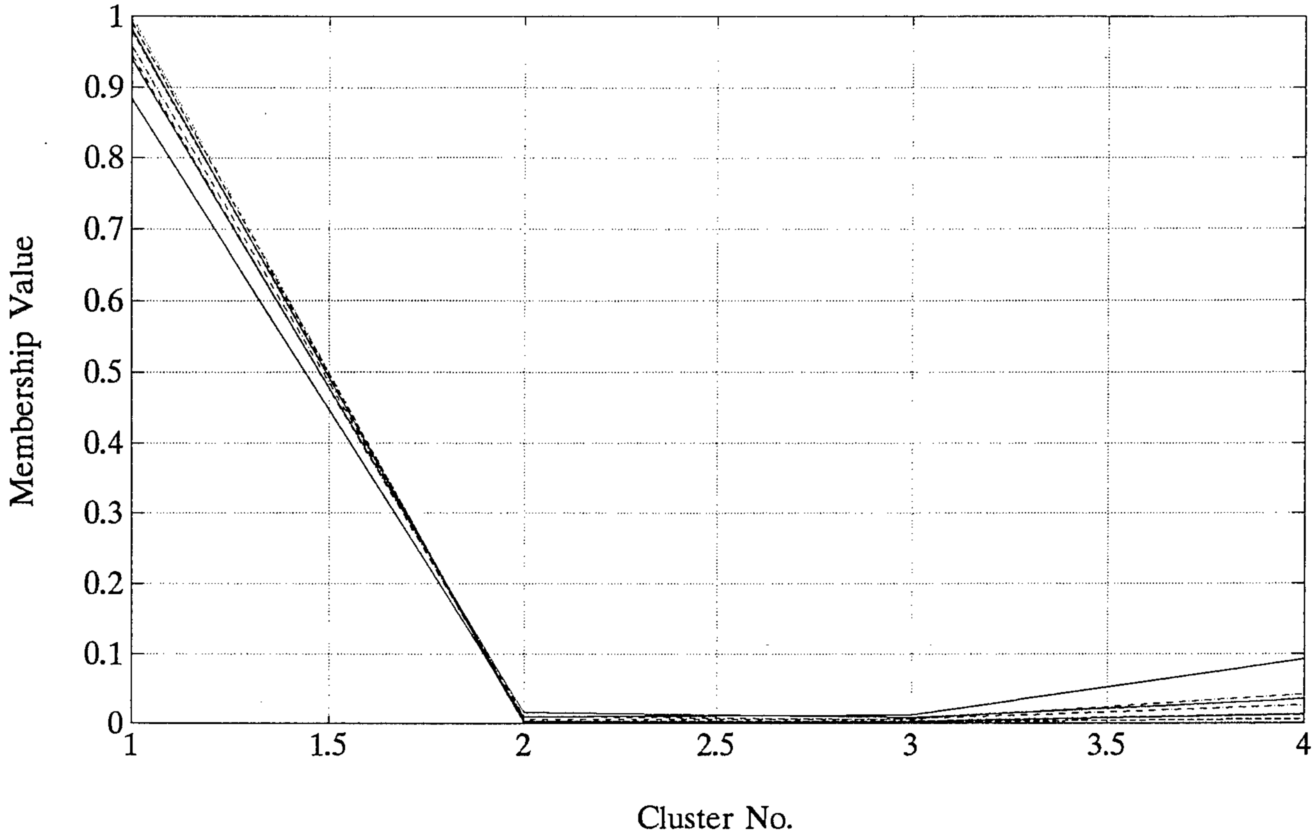

Figure 4.11: Membership values for feature vectors 31-40.

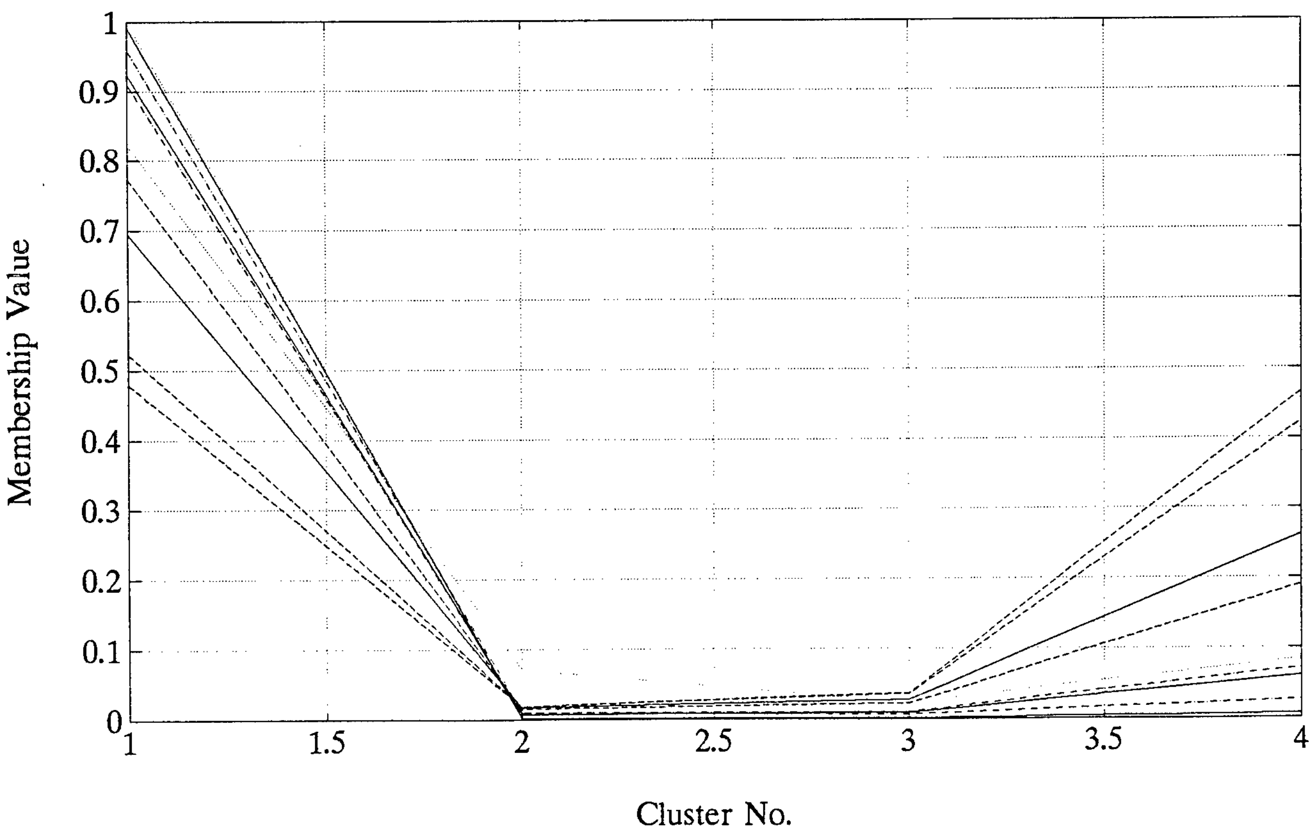

Figure 4.12: Membership values for feature vectors 41-50.

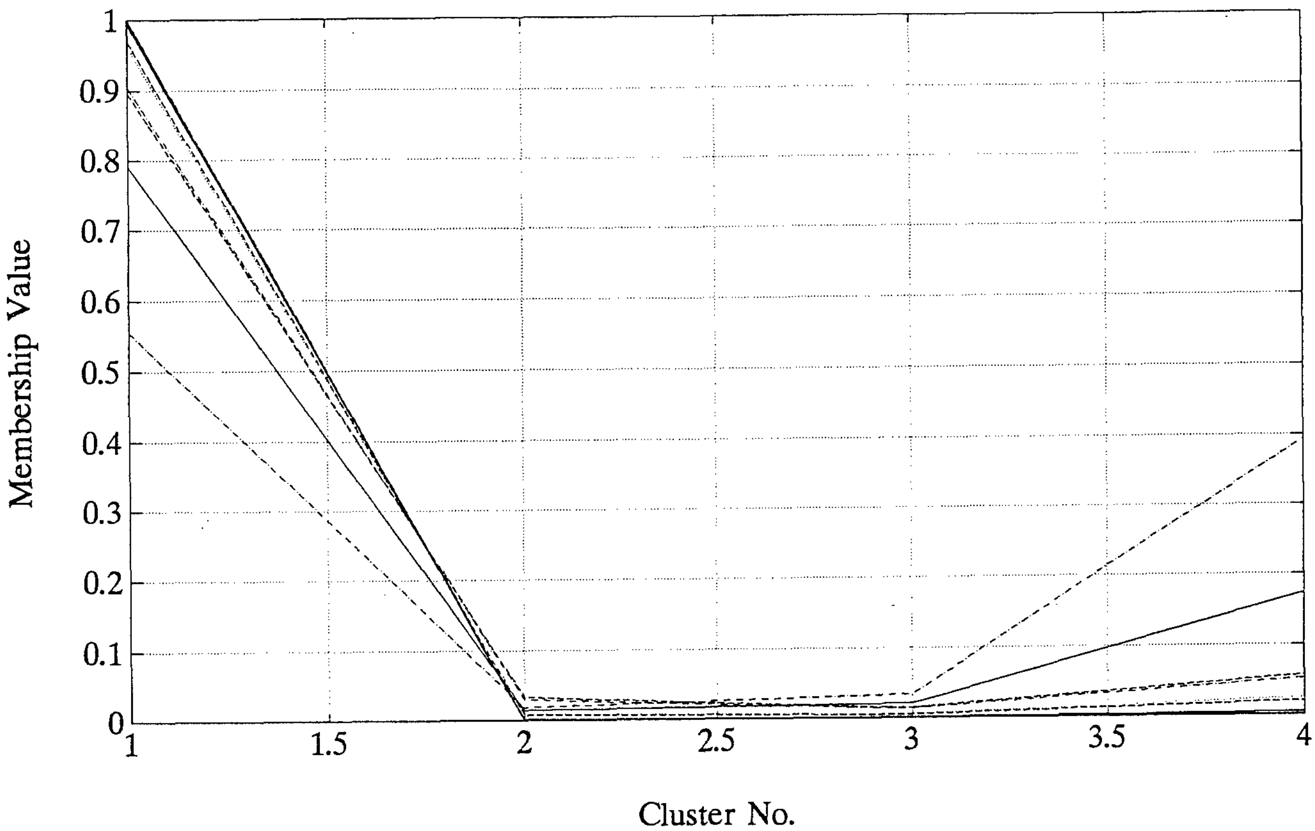

Figure 4.13: Membership values for feature vectors 51-60.

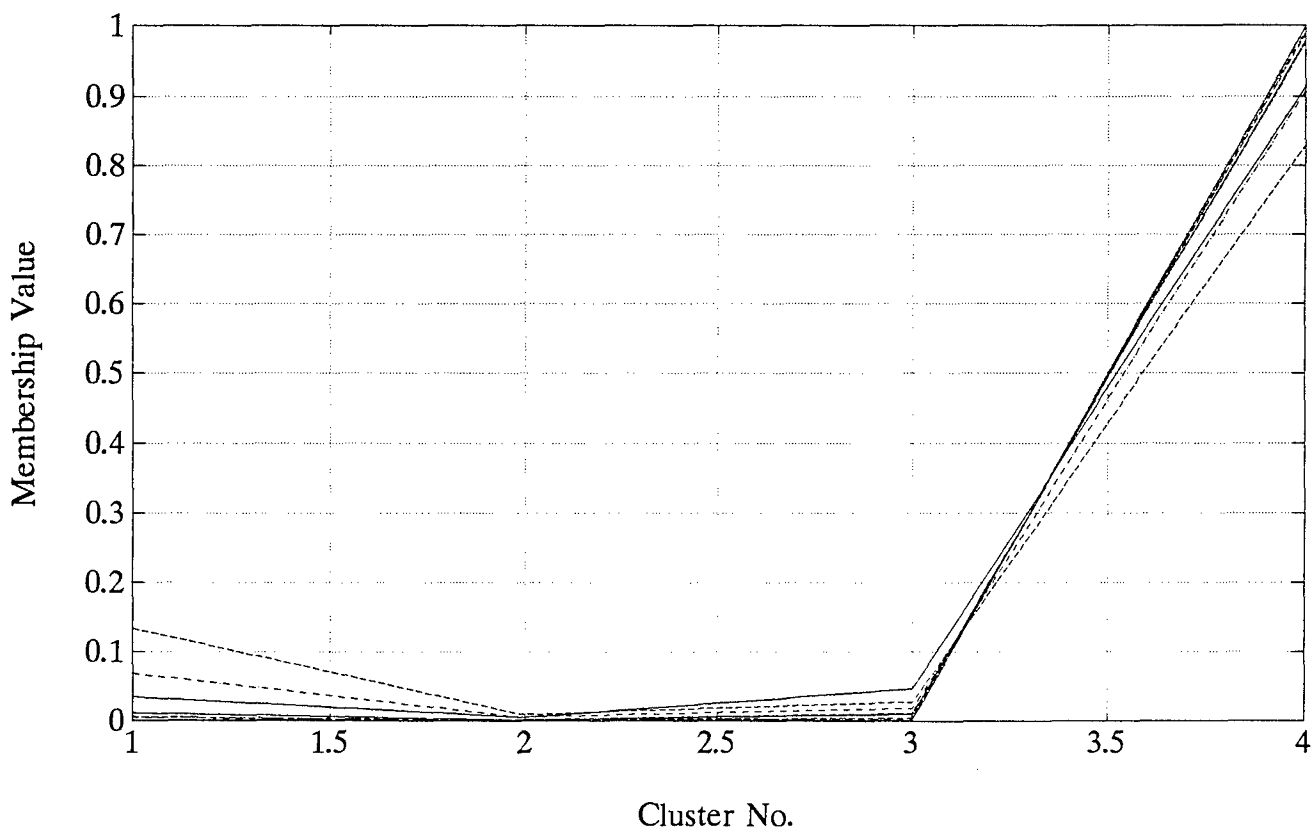

Figure 4.14: Membership values for feature vectors 61-70.

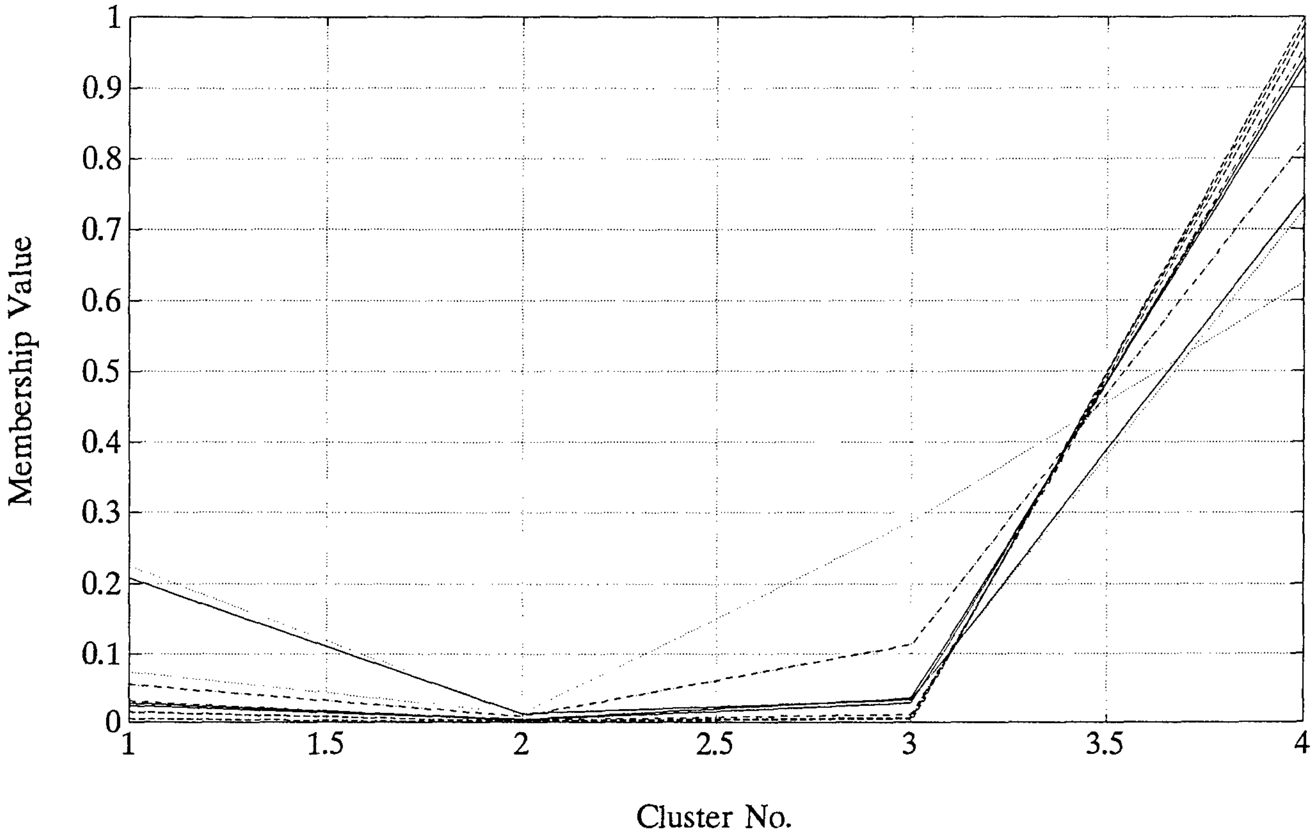

Figure 4.15: Membership values for feature vectors 71-80.

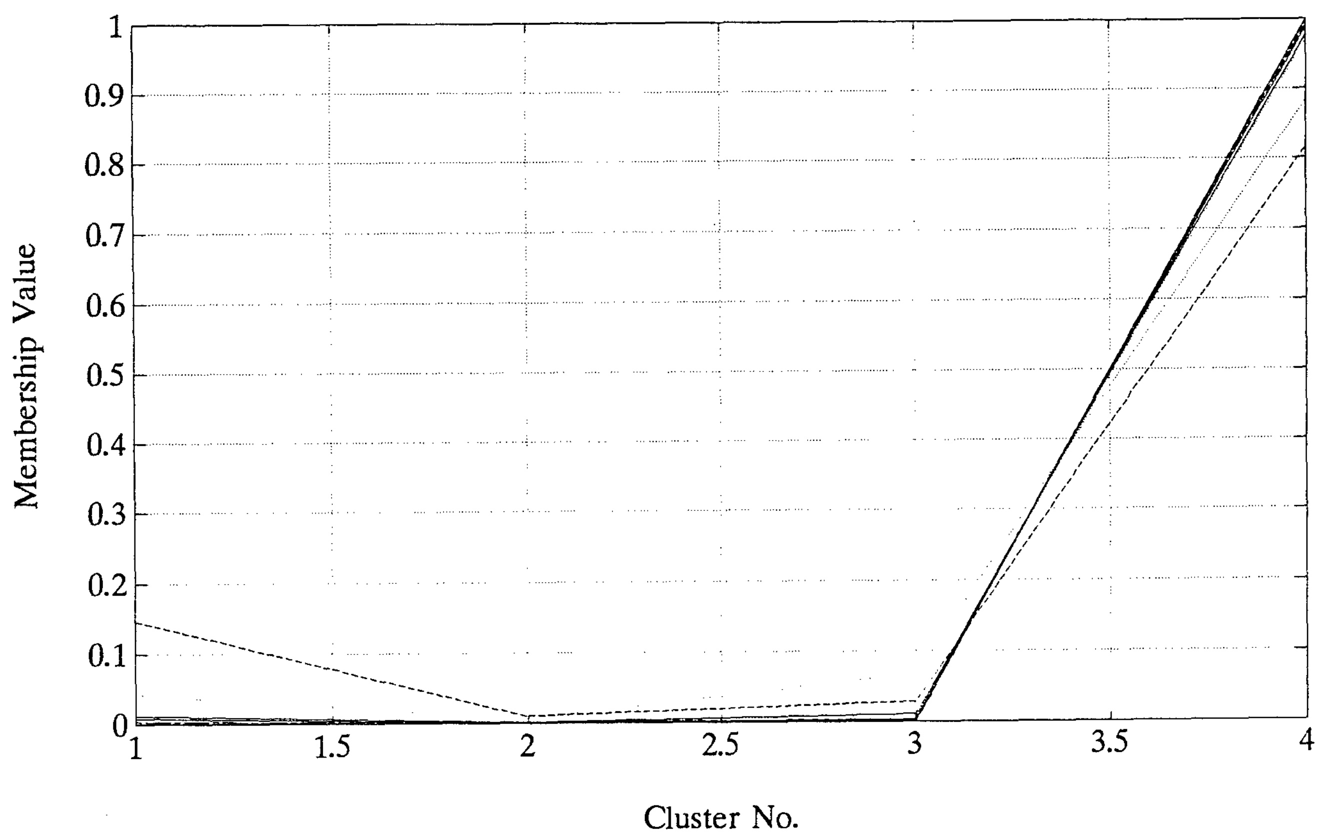

Figure 4.16: Membership values for feature vectors 81-90.

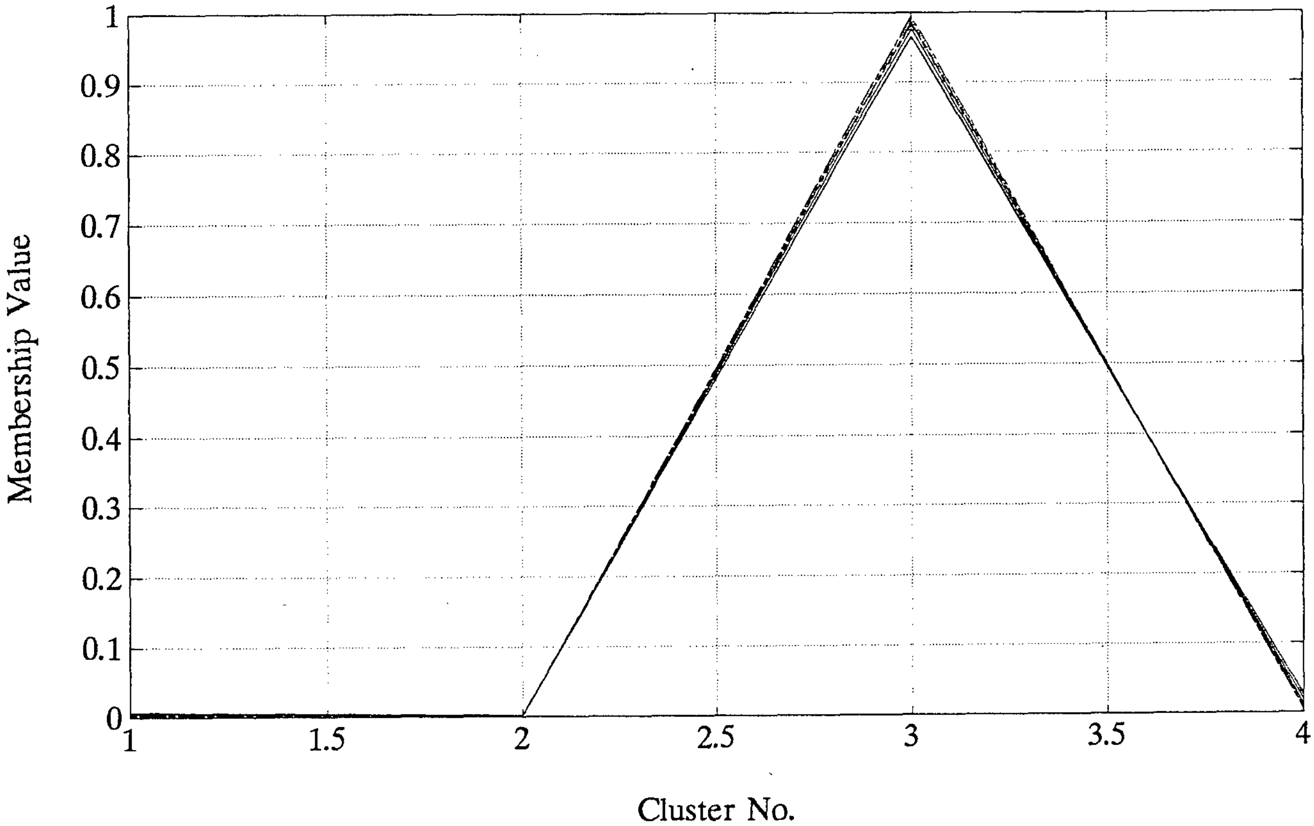

Figure 4.17: Membership values for feature vectors 91-100.

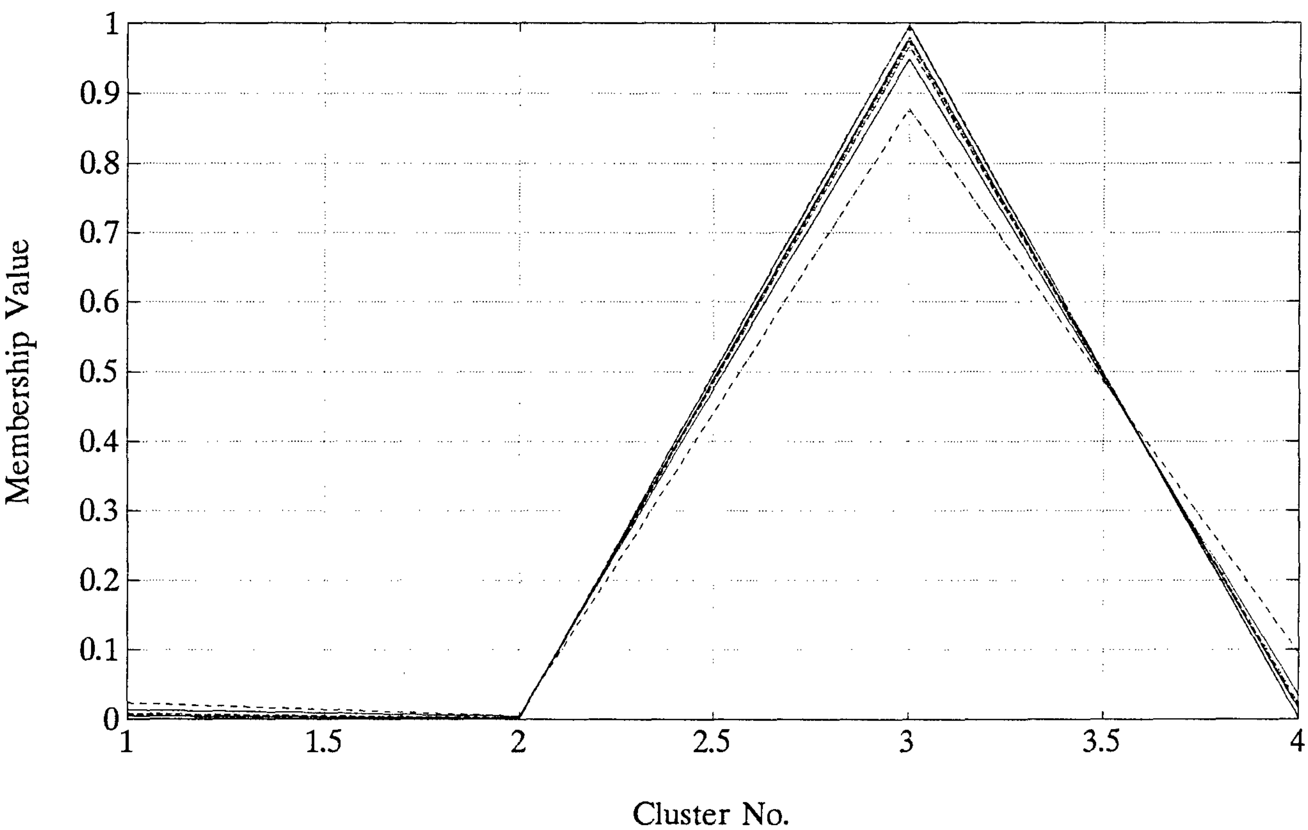

Figure 4.18: Membership values for feature vectors 101-110.

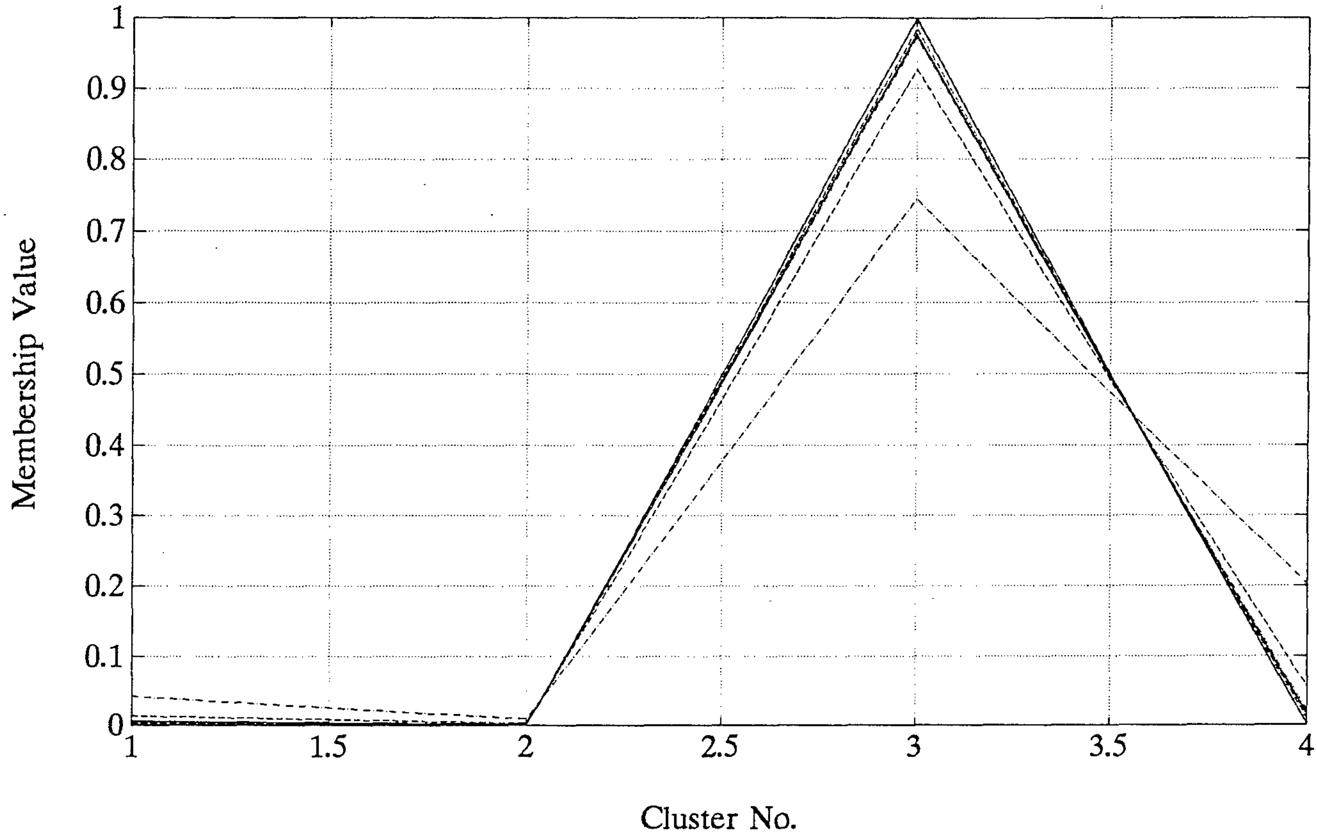

Figure 4.19: Membership values for feature vectors 110-120.

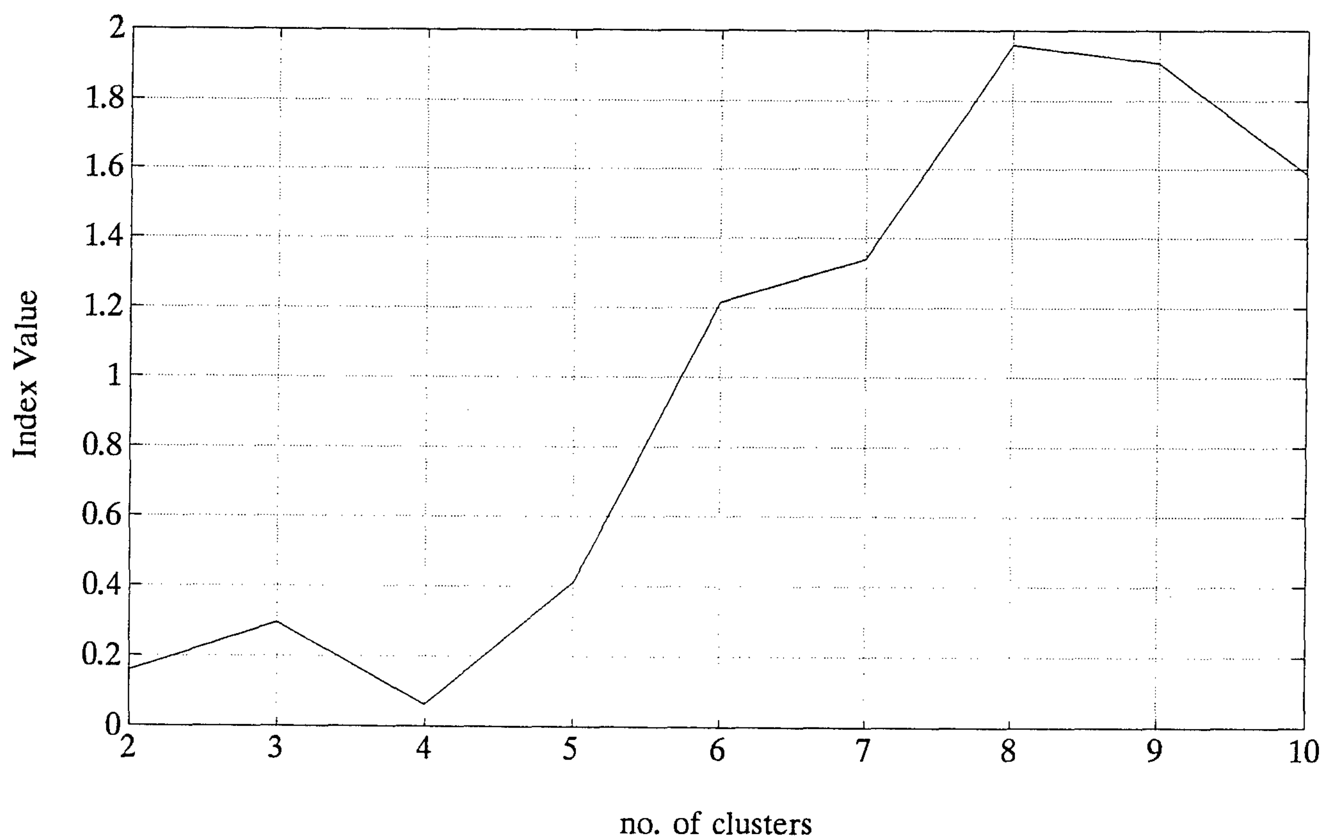

Figure 4.20: Validity measure versus no. of clusters.

Table 4.1: Fuzzy membership matrix U. (There are four rows corresponding to four clusters and one hundred and twenty columns corresponding to the data points.)

Columns 1 through 7

0.0135	0.0077	0.0153	0.0426	0.0063	0.0097	0.0214
0.9764	0.9867	0.9753	0.9328	0.9896	0.9831	0.9656
0.0034	0.0019	0.0030	0.0077	0.0013	0.0024	0.0041
0.0066	0.0037	0.0064	0.0169	0.0027	0.0047	0.0088

Columns 8 through 14

0.0270	0.0039	0.0067	0.0110	0.0065	0.0014	0.0086
0.9568	0.9933	0.9884	0.9810	0.9894	0.9977	0.9850
0.0051	0.0009	0.0016	0.0028	0.0013	0.0003	0.0021
0.0110	0.0019	0.0032	0.0053	0.0028	0.0006	0.0042

Columns 15 through 21

0.0157	0.0019	0.0014	0.0028	0.0025	0.0025	0.0072
0.9725	0.9968	0.9976	0.9952	0.9958	0.9958	0.9876
0.0040	0.0004	0.0003	0.0007	0.0005	0.0005	0.0018
0.0078	0.0008	0.0006	0.0013	0.0011	0.0011	0.0034

Columns 22 through 28

0.0120	0.0268	0.0077	0.0047	0.0678	0.0048	0.0042
0.9790	0.9572	0.9875	0.9919	0.8947	0.9922	0.9928
0.0030	0.0051	0.0016	0.0011	0.0116	0.0010	0.0010
0.0059	0.0109	0.0033	0.0023	0.0259	0.0021	0.0020

Columns 29 through 35

0.0230	0.0509	0.8853	0.9791	0.9998	0.9459	0.9401
0.9593	0.9201	0.0099	0.0046	0.0000	0.0057	0.0163
0.0061	0.0090	0.0125	0.0029	0.0000	0.0063	0.0084
0.0116	0.0200	0.0923	0.0134	0.0001	0.0421	0.0353

Columns 36 through 42

0.9918	0.9985	0.9573	0.9827	0.9799	0.9921	0.5229
0.0011	0.0003	0.0107	0.0022	0.0044	0.0011	0.0182
0.0010	0.0002	0.0060	0.0022	0.0028	0.0010	0.0352
0.0060	0.0011	0.0260	0.0130	0.0129	0.0058	0.4238

Columns 43 through 49

0.8204	0.9100	0.9236	0.4796	0.9994	0.9586	0.6945
0.0717	0.0084	0.0074	0.0180	0.0001	0.0103	0.0168
0.0237	0.0101	0.0087	0.0363	0.0001	0.0058	0.0272
0.0842	0.0716	0.0603	0.4661	0.0004	0.0253	0.2615

Columns 50 through 56

0.7735	0.7916	0.9999	0.9997	0.5545	0.9942	0.8960
0.0148	0.0142	0.0000	0.0001	0.0182	0.0008	0.0333
0.0219	0.0205	0.0000	0.0000	0.0341	0.0007	0.0144
0.1898	0.1738	0.0001	0.0002	0.3932	0.0043	0.0562

Table 4.1 (cont.)

Columns 57 through 63

0.9608	0.9045	0.9999	0.9683	0.0342	0.0059	0.0002
0.0096	0.0297	0.0000	0.0075	0.0048	0.0007	0.0000
0.0055	0.0133	0.0000	0.0044	0.0467	0.0047	0.0001
0.0241	0.0524	0.0001	0.0198	0.9143	0.9888	0.9996

Columns 64 through 70

0.0049	0.0105	0.0118	0.0112	0.0683	0.0004	0.1341
0.0005	0.0013	0.0015	0.0014	0.0052	0.0000	0.0088
0.0023	0.0093	0.0108	0.0101	0.0183	0.0002	0.0271
0.9923	0.9789	0.9760	0.9773	0.9082	0.9994	0.8300

Columns 71 through 77

0.0284	0.0000	0.2256	0.0556	0.0246	0.0155	0.0742
0.0039	0.0000	0.0127	0.0087	0.0033	0.0014	0.0134
0.0352	0.0000	0.0338	0.1126	0.0285	0.0061	0.2873
0.9325	1.0000	0.7279	0.8231	0.9437	0.9770	0.6251

Columns 78 through 84

0.0314	0.2086	0.0052	0.0078	0.1465	0.0195	0.0039
0.0026	0.0121	0.0006	0.0007	0.0094	0.0017	0.0005
0.0106	0.0329	0.0040	0.0035	0.0283	0.0074	0.0029
0.9554	0.7465	0.9902	0.9880	0.8157	0.9715	0.9927

Columns 85 through 91

0.0002	0.0040	0.0428	0.0001	0.0115	0.0009	0.0051
0.0000	0.0004	0.0063	0.0000	0.0014	0.0001	0.0013
0.0001	0.0019	0.0678	0.0001	0.0104	0.0006	0.9758
0.9996	0.9938	0.8832	0.9998	0.9767	0.9985	0.0178

Columns 92 through 98

0.0041	0.0083	0.0030	0.0073	0.0013	0.0020	0.0013
0.0012	0.0025	0.0008	0.0019	0.0004	0.0005	0.0003
0.9832	0.9666	0.9860	0.9647	0.9943	0.9910	0.9942
0.0115	0.0226	0.0102	0.0261	0.0040	0.0066	0.0042

Columns 99 through 105

0.0071	0.0038	0.0043	0.0084	0.0007	0.0231	0.0131
0.0018	0.0011	0.0011	0.0026	0.0002	0.0055	0.0041
0.9654	0.9844	0.9795	0.9661	0.9969	0.8772	0.9483
0.0256	0.0107	0.0150	0.0229	0.0022	0.0942	0.0345

Table 4.1 (cont.)

Columns 106 through 112

0.0063	0.0075	0.0055	0.0008	0.0006	0.0003	0.0145
0.0019	0.0023	0.0016	0.0002	0.0002	0.0001	0.0036
0.9745	0.9697	0.9774	0.9965	0.9976	0.9985	0.9262
0.0174	0.0206	0.0154	0.0025	0.0017	0.0010	0.0557

Columns 113 through 119

0.0040	0.0431	0.0052	0.0035	0.0007	0.0060	0.0057
0.0010	0.0096	0.0013	0.0010	0.0002	0.0016	0.0017
0.9811	0.7445	0.9754	0.9855	0.9968	0.9710	0.9768
0.0138	0.2028	0.0181	0.0100	0.0022	0.0214	0.0158

Column 120

0.0001
0.0000
0.9997
0.0002

Table 4.2: Membership values per feature vector.

olumn (1): Feature Vector No.

olumn (2): Initially assigned cluster assignment.

olumn (3): Fuzzy c-means cluster assignment.

Column (4): Membership value for algorithm derived cluster 1.

Column (5): Membership value for algorithm derived cluster 2.

Column (6): Membership value for algorithm derived cluster 3.

Column (7): Membership value for algorithm derived cluster 4.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.0000	1.0000	2.0000	0.0135	0.9764	0.0034	0.0066
2.0000	1.0000	2.0000	0.0077	0.9867	0.0019	0.0037
3.0000	1.0000	2.0000	0.0153	0.9753	0.0030	0.0064
4.0000	1.0000	2.0000	0.0426	0.9328	0.0077	0.0169
5.0000	1.0000	2.0000	0.0063	0.9896	0.0013	0.0027
6.0000	1.0000	2.0000	0.0097	0.9831	0.0024	0.0047
7.0000	1.0000	2.0000	0.0214	0.9656	0.0041	0.0088
8.0000	1.0000	2.0000	0.0270	0.9568	0.0051	0.0110
9.0000	1.0000	2.0000	0.0039	0.9933	0.0009	0.0019
10.0000	1.0000	2.0000	0.0067	0.9884	0.0016	0.0032
11.0000	1.0000	2.0000	0.0110	0.9810	0.0028	0.0053
12.0000	1.0000	2.0000	0.0065	0.9894	0.0013	0.0028
13.0000	1.0000	2.0000	0.0014	0.9977	0.0003	0.0006
14.0000	1.0000	2.0000	0.0086	0.9850	0.0021	0.0042
15.0000	1.0000	2.0000	0.0157	0.9725	0.0040	0.0078
16.0000	1.0000	2.0000	0.0019	0.9968	0.0004	0.0008
17.0000	1.0000	2.0000	0.0014	0.9976	0.0003	0.0006
18.0000	1.0000	2.0000	0.0028	0.9952	0.0007	0.0013
19.0000	1.0000	2.0000	0.0025	0.9958	0.0005	0.0011
20.0000	1.0000	2.0000	0.0025	0.9958	0.0005	0.0011
21.0000	1.0000	2.0000	0.0072	0.9876	0.0018	0.0034
22.0000	1.0000	2.0000	0.0120	0.9790	0.0030	0.0059
23.0000	1.0000	2.0000	0.0268	0.9572	0.0051	0.0109
24.0000	1.0000	2.0000	0.0077	0.9875	0.0016	0.0033
25.0000	1.0000	2.0000	0.0047	0.9919	0.0011	0.0023
26.0000	1.0000	2.0000	0.0678	0.8947	0.0116	0.0259
27.0000	1.0000	2.0000	0.0048	0.9922	0.0010	0.0021
28.0000	1.0000	2.0000	0.0042	0.9928	0.0010	0.0020
29.0000	1.0000	2.0000	0.0230	0.9593	0.0061	0.0116
30.0000	1.0000	2.0000	0.0509	0.9201	0.0090	0.0200
31.0000	2.0000	1.0000	0.8853	0.0099	0.0125	0.0923
32.0000	2.0000	1.0000	0.9791	0.0046	0.0029	0.0134
33.0000	2.0000	1.0000	0.9998	0.0000	0.0000	0.0001
34.0000	2.0000	1.0000	0.9459	0.0057	0.0063	0.0421
35.0000	2.0000	1.0000	0.9401	0.0163	0.0084	0.0353
36.0000	2.0000	1.0000	0.9918	0.0011	0.0010	0.0060
37.0000	2.0000	1.0000	0.9985	0.0003	0.0002	0.0011
38.0000	2.0000	1.0000	0.9573	0.0107	0.0060	0.0260
39.0000	2.0000	1.0000	0.9827	0.0022	0.0022	0.0130
40.0000	2.0000	1.0000	0.9799	0.0044	0.0028	0.0129
41.0000	2.0000	1.0000	0.9921	0.0011	0.0010	0.0058
42.0000	2.0000	1.0000	0.5229	0.0182	0.0352	0.4238
43.0000	2.0000	1.0000	0.8204	0.0717	0.0237	0.0842
44.0000	2.0000	1.0000	0.9100	0.0084	0.0101	0.0716
45.0000	2.0000	1.0000	0.9236	0.0074	0.0087	0.0603
46.0000	2.0000	1.0000	0.4796	0.0180	0.0363	0.4661
47.0000	2.0000	1.0000	0.9994	0.0001	0.0001	0.0004
48.0000	2.0000	1.0000	0.9586	0.0103	0.0058	0.0253
49.0000	2.0000	1.0000	0.6945	0.0168	0.0272	0.2615
50.0000	2.0000	1.0000	0.7735	0.0148	0.0219	0.1898

Table 4.2 (cont.)

51.0000	2.0000	1.0000	0.7916	0.0142	0.0205	0.1738
52.0000	2.0000	1.0000	0.9999	0.0000	0.0000	0.0001
53.0000	2.0000	1.0000	0.9997	0.0001	0.0000	0.0002
54.0000	2.0000	1.0000	0.5545	0.0182	0.0341	0.3932
55.0000	2.0000	1.0000	0.9942	0.0008	0.0007	0.0043
56.0000	2.0000	1.0000	0.8960	0.0333	0.0144	0.0562
57.0000	2.0000	1.0000	0.9608	0.0096	0.0055	0.0241
58.0000	2.0000	1.0000	0.9045	0.0297	0.0133	0.0524
59.0000	2.0000	1.0000	0.9999	0.0000	0.0000	0.0001
60.0000	2.0000	1.0000	0.9683	0.0075	0.0044	0.0198
61.0000	3.0000	4.0000	0.0342	0.0048	0.0467	0.9143
62.0000	3.0000	4.0000	0.0059	0.0007	0.0047	0.9888
63.0000	3.0000	4.0000	0.0002	0.0000	0.0001	0.9996
64.0000	3.0000	4.0000	0.0049	0.0005	0.0023	0.9923
65.0000	3.0000	4.0000	0.0105	0.0013	0.0093	0.9789
66.0000	3.0000	4.0000	0.0118	0.0015	0.0108	0.9760
67.0000	3.0000	4.0000	0.0112	0.0014	0.0101	0.9773
68.0000	3.0000	4.0000	0.0683	0.0052	0.0183	0.9082
69.0000	3.0000	4.0000	0.0004	0.0000	0.0002	0.9994
70.0000	3.0000	4.0000	0.1341	0.0088	0.0271	0.8300
71.0000	3.0000	4.0000	0.0284	0.0039	0.0352	0.9325
72.0000	3.0000	4.0000	0.0000	0.0000	0.0000	1.0000
73.0000	3.0000	4.0000	0.2256	0.0127	0.0338	0.7279
74.0000	3.0000	4.0000	0.0556	0.0087	0.1126	0.8231
75.0000	3.0000	4.0000	0.0246	0.0033	0.0285	0.9437
76.0000	3.0000	4.0000	0.0155	0.0014	0.0061	0.9770
77.0000	3.0000	4.0000	0.0742	0.0134	0.2873	0.6251
78.0000	3.0000	4.0000	0.0314	0.0026	0.0106	0.9554
79.0000	3.0000	4.0000	0.2086	0.0121	0.0329	0.7465
80.0000	3.0000	4.0000	0.0052	0.0006	0.0040	0.9902
81.0000	3.0000	4.0000	0.0078	0.0007	0.0035	0.9880
82.0000	3.0000	4.0000	0.1465	0.0094	0.0283	0.8157
83.0000	3.0000	4.0000	0.0195	0.0017	0.0074	0.9715
84.0000	3.0000	4.0000	0.0039	0.0005	0.0029	0.9927
85.0000	3.0000	4.0000	0.0002	0.0000	0.0001	0.9996
86.0000	3.0000	4.0000	0.0040	0.0004	0.0019	0.9938
87.0000	3.0000	4.0000	0.0428	0.0063	0.0678	0.8832
88.0000	3.0000	4.0000	0.0001	0.0000	0.0001	0.9998
89.0000	3.0000	4.0000	0.0115	0.0014	0.0104	0.9767
90.0000	3.0000	4.0000	0.0009	0.0001	0.0006	0.9985
91.0000	4.0000	3.0000	0.0051	0.0013	0.9758	0.0178
92.0000	4.0000	3.0000	0.0041	0.0012	0.9832	0.0115
93.0000	4.0000	3.0000	0.0083	0.0025	0.9666	0.0226
94.0000	4.0000	3.0000	0.0030	0.0008	0.9860	0.0102
95.0000	4.0000	3.0000	0.0073	0.0019	0.9647	0.0261
96.0000	4.0000	3.0000	0.0013	0.0004	0.9943	0.0040
97.0000	4.0000	3.0000	0.0020	0.0005	0.9910	0.0066
98.0000	4.0000	3.0000	0.0013	0.0003	0.9942	0.0042
99.0000	4.0000	3.0000	0.0071	0.0018	0.9654	0.0256
100.0000	4.0000	3.0000	0.0038	0.0011	0.9844	0.0107

101.0000	4.0000	3.0000	0.0043	0.0011	0.9795	0.0150
102.0000	4.0000	3.0000	0.0084	0.0026	0.9661	0.0229
103.0000	4.0000	3.0000	0.0007	0.0002	0.9969	0.0022
104.0000	4.0000	3.0000	0.0231	0.0055	0.8772	0.0942
105.0000	4.0000	3.0000	0.0131	0.0041	0.9483	0.0345
106.0000	4.0000	3.0000	0.0063	0.0019	0.9745	0.0174
107.0000	4.0000	3.0000	0.0075	0.0023	0.9697	0.0206
108.0000	4.0000	3.0000	0.0055	0.0016	0.9774	0.0154
109.0000	4.0000	3.0000	0.0008	0.0002	0.9965	0.0025
110.0000	4.0000	3.0000	0.0006	0.0002	0.9976	0.0017
111.0000	4.0000	3.0000	0.0003	0.0001	0.9985	0.0010
112.0000	4.0000	3.0000	0.0145	0.0036	0.9262	0.0557
113.0000	4.0000	3.0000	0.0040	0.0010	0.9811	0.0138
114.0000	4.0000	3.0000	0.0431	0.0096	0.7445	0.2028
115.0000	4.0000	3.0000	0.0052	0.0013	0.9754	0.0181
116.0000	4.0000	3.0000	0.0035	0.0010	0.9855	0.0100
117.0000	4.0000	3.0000	0.0007	0.0002	0.9968	0.0022
118.0000	4.0000	3.0000	0.0060	0.0016	0.9710	0.0214
119.0000	4.0000	3.0000	0.0057	0.0017	0.9768	0.0158
120.0000	4.0000	3.0000	0.0001	0.0000	0.9997	0.0002

Chapter 5

Results from Real Data

Real data were obtained from the National Study Center for Trauma and EMS at the University of Maryland. Trianalytics, Inc. maintains the data base for the University of Maryland. The data consist of 200 records corresponding to 100 penetrating (gunshot) wound records for male patients who survived and 100 penetrating (gunshot) wound records for male patients who did not survive. The hospital stay for 97 out of the 100 patients who did not survive was 0 to 1 days indicative of the fact that their death was a direct result of their trauma injuries. The patient population age is around 25-30 years. Also the patients had no preexisting conditions.

Four features were selected to correspond to the variables encountered in field directed trauma score indices. The features, with their encoded severity are:

- **Eye Opening.** 4 = Spontaneous, 3 = To Voice, 2 = To Pain, 1 = None.
- **Verbal Response.** 5 = Oriented, 4 = Confused, 3 = Inappropriate Words, 2 = Incomprehensible Sounds, 1 = No Verbal Response.
- **Motor Response.** 6 = Obeys Command, 5 = Localizes Pain, 4 = Withdraws, 3 = Flexion Response, 2 = Extension Response, 1 = No Motor Response.

- **Is Patient's Respiratory Rate Controlled by Bagging or Ventilator?.** 1 = Yes, 2 = No.

- Missing values are coded as 9s.

Table 5.1 lists the 100 surviving patients records with the corresponding feature values. Table 5.2 lists the 100 nonsurviving patients and their corresponding feature values. The Xie and Beni algorithm was then exercised on the set of the 100 surviving patients to ascertain a reasonable number of clusters to partition the data in. The Xie and Beni cluster validity measure values for 2,3,4,5,6,7,8 and 9 clusters are 0.0421, 0.0144, 0.0466, 0.0123, 0.1495, 0.0912, 0.0714 and 0.0705 respectively. They are plotted in Figure 5.1. The minimum value is 0.0123 and occurs for a partition of 5 clusters. The fuzzy c-means algorithm is next invoked with a 5 clusters partition. The resulting feature vector contents of the 5 clusters are shown in Table 5.3. It is noted that the clustered vectors are intuitively reasonable. The application of the Xie and Beni algorithm to the 100 nonsurviving patients yields the cluster validity measure values 0.0522, 0.1333, 0.0610, 0.1038, ∞, 0.3901, 0.3211, and 0.1570 for 2,3,4,5,6,7,8 and 9 clusters respectively. The ∞ value is the result of occasional singularities in the algorithm when two cluster centers coincide. They are plotted in Figure 5.2. Upon inspection of the results, in this case, we selected the 3 clusters configuration over the 2 clusters minimum solution as being more representative of the data structure. This is indicative of the fact that the cluster algorithms provide useful direction but no guarantee of absolute goal accomplishment.The resulting feature vector contents of the 3 clusters for the nonsurviving class are shown in Table 5.4

For the surviving class of patients nine GPFUs per cluster were then established for each of the 5 clusters with the centering GPFU at the cluster center. The desired GPFN integer characterization for the surviving patients class was set equal to 1 and for the nonsurviving patients to -1. The training algorithm was iterated 500 times with the iteration error results as shown below:

$$\begin{array}{ll}
Initial\ Error : 5.4380 & Error\ after\ 500\ iterations : 0.3331 \\
Initial\ Error : 5.7328 & Error\ after\ 500\ iterations : 0.0000 \\
Initial\ Error : 5.7938 & Error\ after\ 500\ iterations : 0.0441 \\
Initial\ Error : 6.9660 & Error\ after\ 500\ iterations : 0.0000 \\
Initial\ Error : 7.1010 & Error\ after\ 500\ iterations : 0.0833
\end{array} \tag{5.1}$$

A similar, 500 iteration training phase was executed for the nonsurviving patients 3 clusters with the training error history results as follows:

$$\begin{array}{ll}
Initial\ Error : 0.3857 & Error\ after\ 500\ iterations : 0.2049 \\
Initial\ Error : 0.4422 & Error\ after\ 500\ iterations : 0.0000 \\
Initial\ Error : 0.5143 & Error\ after\ 500\ iterations : 0.0536
\end{array} \tag{5.2}$$

The classification performance for the set of data representing Classes 1 (Surviving Patients) and 1 (Nonsurviving Patients) is established as follows. The training phase of the classifier created two sets of GPFNs. One for Class 1 and one for Class 2. A data point that belongs to Class 1 must ideally yield a value of 1 while a data point that belongs to Class 2 must yield the value -1. There are 200 data points to consider, 100 from Class 1 and 100 from Class 2. Each point is fed to the GPFN corresponding to Class 1 and the GPFN corresponding to Class 2. Two responses are thus noted. Next, the percent deviation of the actual response from the desired response (the desired response is 1 for Class 1 and -1 for Class 2) is calculated and the data point is assigned to the class with the smallest percent deviation. The results for the 200 points are given in Table 5.5. It is noted that the feature vectors are listed in this Table as grouped in clusters. Thus, the first three entries correspond to the first cluster of the surviving patients, the next four entries to the second cluster of the surviving patients, etc. Records 100 through 200 correspond to the nonsurviving patients. Column (2) is the known correct classification, column (3) the calculated classification, column (4) the response of the Class 1 GPFN, column (5)

the percent error resulting from the Class 1 GPFN response, column (6) the response of the Class 2 GPFN and column (7) the percent error resulting from the Class 2 response. Comparing columns (1) and (2) of Table 5.5 we note that the correct classification rate is 88.5%. This is not unanticipated because the data distributions from the two classes are overlapping.

5.1 Direct Classification Encoding

The above classification results bring forward the probabilistic nature of the classification problem. It is ideally desired that features be selected that effect a complete and unambiguous separation of the various classes in feature space. However, most real problems involve inescapable feature vector overlaps meaning that the same feature vector is observed for members of different classes. In this case, assignment to a certain class is effected by probabilistic arguments (Bayes Theorem) that basically select the most likely class for this feature vector as demonstrated by experience or theoretical considerations. To emphasize this case we encoded through a GPFU of unit variance each entry from the surviving class of patients and added this group of gaussians in feature space. We did the same for the 100 patients of the nonsurviving class. Each one of these surfaces was then used to compute a score for each patient which was, in turn, assigned to the class that exhibited the highest score. The results are shown in Table 5.6. The correct classification rate is 86.5 %. A careful scrutiny of the misclassified cases reveals the following:

Nonsurvivors classified as surviving.

- Nonsurviving patients 102, 130, 162, 166, 167, 197 and 199 are classified as surviving. Their feature vector is (4,5,6,2). There are 65 such vectors in the surviving category versus 8 in the nonsurviving category.
- Nonsurviving patients 108, 111, 112, 117, 118, 119, 121, 123, 124, 135 and 194 are classified as surviving. Their feature vector is (9,9,9,1). There are 11 such vectors in

the nonsurviving category versus 1 in the surviving category. However, there are 17 (9,9,9,2) vectors in the surviving category versus 1 in the nonsurviving category.

- Nonsurviving patients 133, 136 and 171 are classified as surviving. Their feature vector is (9,9,9,2). There are 17 such vectors in the surviving category and one in the nonsurviving category.
- Nonsurviving patient 173 is classified as surviving. His feature vector is (3,4,5,2). There is one such vector in the nonsurviving category and one in the surviving category. However the score is much higher for the surviving category due to the influence of the (4,5,6,2) vectors.

Survivors classified as nonsurviving.

- Surviving patients 23, 62 and 78 are classified as nonsurviving. Their feature vector is (1,1,1,1). There are 48 such vectors in the nonsurviving category versus 3 in the surviving category.
- Surviving patient 42 is classified as nonsurviving. His feature vector is (9,9,9,9). There are 13 such vectors in the nonsurviving category versus 1 in the surviving category.
- Surviving patient 100 is classified as nonsurviving. His feature vector is (1,1,2,1). There is one such vector in the nonsurviving category and one in the surviving category. However the score is much higher for the nonsurviving category due to the influence of the (1,1,1,1) vectors.

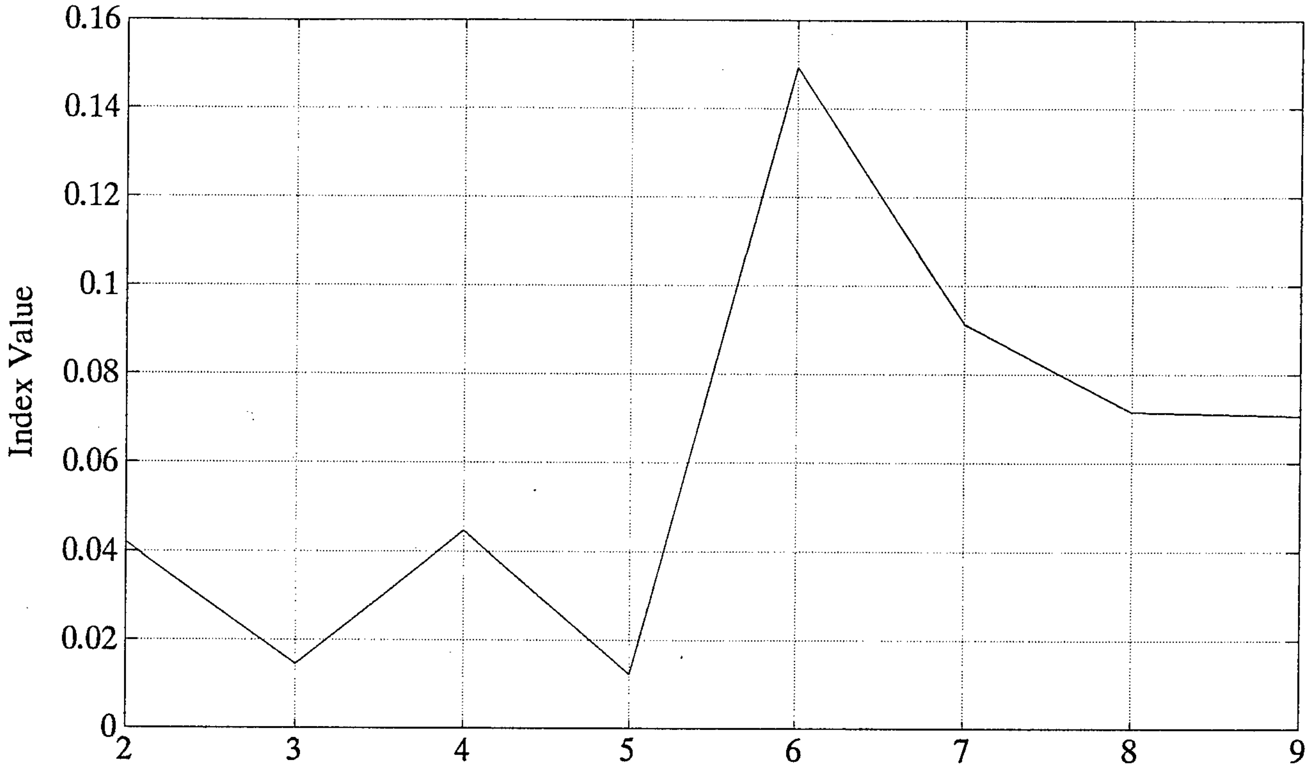

Figure 5.1: Validity Measure versus No. of Clusters for Surviving Patients

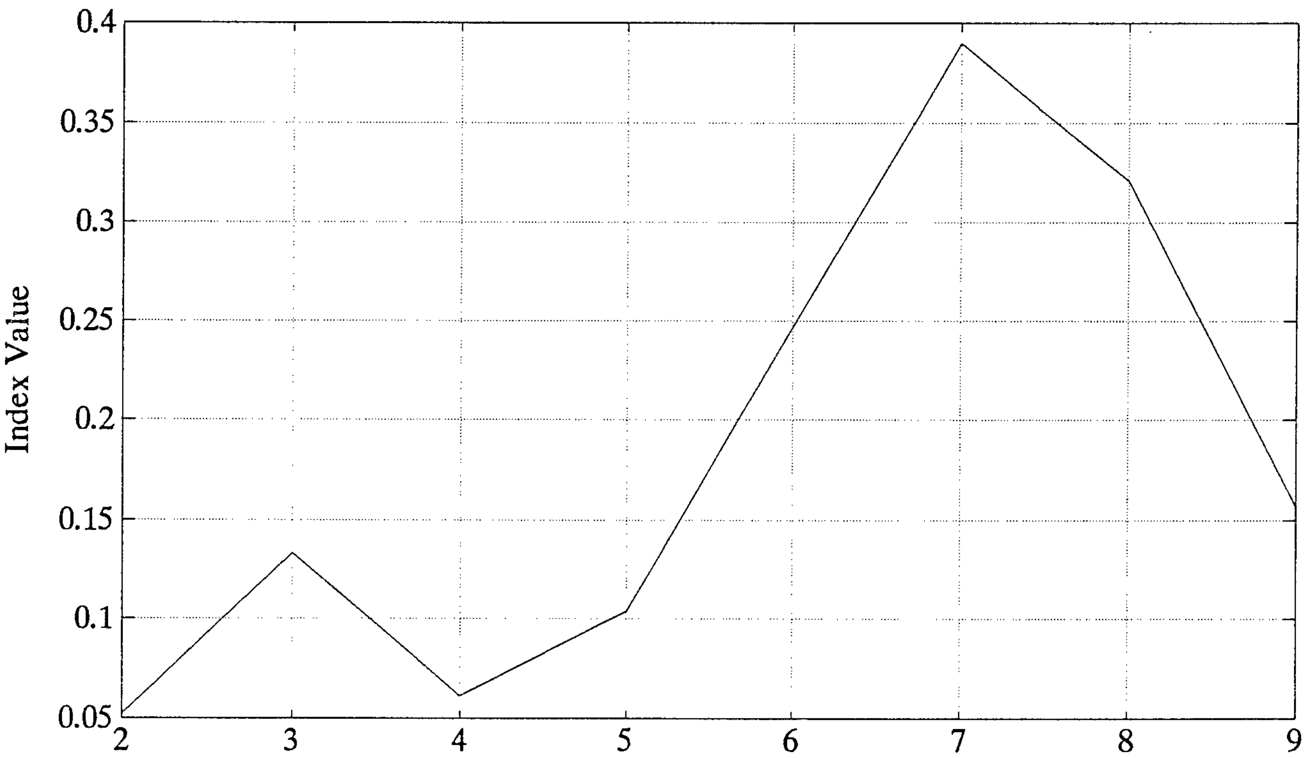

Figure 5.2: Validity Measure versus No. of Clusters for Nonsurviving Patients

Table 5.1: Surviving Patients Feature Vectors.

(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
1	4	5	6	2	26	4	5	6	2
2	3	4	5	2	27	4	5	6	2
3	4	5	6	2	28	4	5	6	2
4	4	1	6	1	29	9	9	9	2
5	4	5	6	2	30	9	9	9	2
6	4	5	6	2	31	9	9	9	2
7	4	5	6	2	32	3	5	6	2
8	4	1	6	2	33	4	5	6	2
9	4	5	6	2	34	4	5	6	2
10	4	5	6	2	35	4	5	6	2
11	4	5	6	2	36	4	5	6	2
12	4	5	6	2	37	4	5	6	2
13	4	5	6	2	38	4	5	6	2
14	4	5	6	1	39	4	5	6	2
15	4	5	6	2	40	9	9	9	2
16	4	5	6	2	41	9	9	9	2
17	4	5	6	2	42	9	9	9	9
18	9	9	9	2	43	4	5	6	2
19	9	9	9	2	44	4	5	6	2
20	9	9	9	2	45	9	9	9	2
21	4	5	6	2	46	4	5	6	2
22	9	9	9	2	47	4	5	6	2
23	1	1	1	1	48	9	9	9	2
24	4	5	6	2	49	4	5	6	2
25	9	9	9	2	50	4	5	6	2

Column (1): Patient No.

Column (2): Eye Opening.

Column (3): Verbal Response.

Column (4): Motor Response .

Column (5): Respiratory Assistance.

Table 5.1 (cont.)

(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
51	4	5	6	2	76	4	5	6	2
52	4	5	6	2	77	4	5	6	2
53	4	5	6	2	78	1	1	1	1
54	9	9	9	2	79	4	5	6	2
55	4	5	6	2	80	4	5	6	2
56	4	5	6	2	81	4	5	6	2
57	4	5	6	2	82	4	5	6	2
58	4	5	6	2	83	9	9	9	2
59	4	5	6	2	84	4	5	6	2
60	4	5	6	2	85	9	9	9	2
61	9	9	9	2	86	4	5	6	2
62	1	1	1	1	87	4	5	6	2
63	4	5	6	2	88	4	5	6	1
64	9	9	9	1	89	4	5	6	2
65	9	9	9	2	90	9	9	9	2
66	4	5	6	2	91	4	5	6	2
67	4	5	6	2	92	4	5	6	2
68	4	5	6	2	93	4	5	6	2
69	9	9	9	2	94	9	9	9	2
70	9	9	9	1	95	4	5	6	2
71	9	9	9	2	96	4	5	6	2
72	9	9	9	2	97	4	5	6	2
73	4	5	6	2	98	4	5	6	2
74	4	5	6	2	99	4	5	6	2
75	4	5	6	2	100	1	1	2	1

Table 5.2: Nonsurviving Patients Feature Vectors.

(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
1	1	1	1	1	26	9	9	9	9
2	4	5	6	2	27	1	1	1	1
3	1	1	1	1	28	1	1	1	1
4	9	9	9	9	29	1	1	1	2
5	1	1	1	1	30	4	5	6	2
6	1	1	1	1	31	1	1	1	9
7	1	1	1	1	32	9	9	9	9
8	9	9	9	1	33	9	9	9	2
9	1	1	1	1	34	1	1	1	1
10	1	1	1	1	35	9	9	9	1
11	9	9	9	1	36	9	9	9	2
12	9	9	9	1	37	1	1	1	1
13	1	1	1	1	38	1	1	1	2
14	1	1	1	1	39	9	9	9	9
15	1	1	2	1	40	1	1	1	1
16	1	1	1	1	41	1	1	1	1
17	9	9	9	1	42	9	9	9	9
18	9	9	9	1	43	9	9	9	9
19	9	9	9	1	44	1	1	1	1
20	9	9	9	9	45	1	1	1	1
21	9	9	9	1	46	1	1	1	1
22	9	9	9	9	47	1	1	1	1
23	9	9	9	1	48	1	1	1	1
24	9	9	9	1	49	1	1	1	1
25	9	9	9	9	50	1	1	1	1

Column (1): Patient No.
Column (2): Eye Opening.
Column (3): Verbal Response.
Column (4): Motor Response .
Column (5): Respiratory Assistance.

Table 5.2 (cont.)

(1)	(2)	(3)	(4)	(5)
51	1	1	1	1
52	1	1	1	1
53	1	1	1	1
54	1	1	1	1
55	1	1	1	1
56	1	1	1	1
57	1	1	1	1
58	1	1	1	1
59	1	1	1	1
60	1	1	1	1
61	1	1	1	1
62	4	5	6	2
63	3	3	5	9
64	1	1	1	1
65	3	4	4	2
66	4	5	6	2
67	4	5	6	2
68	1	1	1	1
69	3	3	4	2
70	1	1	1	1
71	9	9	9	2
72	1	1	1	1
73	3	4	5	2
74	4	9	9	1
75	1	1	1	1

(1)	(2)	(3)	(4)	(5)
76	1	1	1	1
77	1	1	1	1
78	1	1	1	1
79	1	1	1	1
80	1	1	1	1
81	1	1	1	1
82	4	3	6	1
83	9	9	9	9
84	1	1	1	1
85	9	9	9	9
86	1	1	1	1
87	1	1	1	1
88	9	9	9	9
89	1	1	1	1
90	1	1	1	9
91	9	9	9	9
92	1	1	1	1
93	1	1	1	1
94	9	9	9	1
95	1	1	1	1
96	1	1	1	1
97	4	5	6	2
98	9	9	9	9
99	4	5	6	2
100	4	5	6	9

Table 5.3: Clusters for Surviving Class.

Cluster 1 (3 elements)

3	4	5	2
4	1	6	1
4	1	6	2

Surviving Patients

Table 5.3 (cont.)

Cluster 2 (4 elements)

1	1	1	1
1	1	1	1
1	1	1	1
1	1	2	1

Surviving Patients

Table 5.3 (cont.)

Cluster 3 (68 elements)

4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	1	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
3	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	1
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2
4	5	6	2	4	5	6	2

Surviving Patients

Table 5.3 (cont.)

Cluster 4 (1 element)

9 9 9 9

Surviving Patients

Table 5.3 (cont.)

Cluster 5 (24 elements)

9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	1
9	9	9	2
9	9	9	2
9	9	9	1
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2
9	9	9	2

Surviving Patients

Table 5.4: Clusters for Nonsurviving Class.

Cluster 1 (16 elements)

4	5	6	2
4	5	6	2
1	1	1	9
4	5	6	2
3	3	5	9
3	4	4	2
4	5	6	2
4	5	6	2
3	3	4	2
3	4	5	2
4	9	9	1
4	3	6	1
1	1	1	9
4	5	6	2
4	5	6	2
4	5	6	9

Nonsurviving Patients

Table 5.4 (cont.)

Cluster 2 (28 elements)

9	9	9	9
9	9	9	1
9	9	9	1
9	9	9	1
9	9	9	1
9	9	9	1
9	9	9	1
9	9	9	9
9	9	9	1
9	9	9	9
9	9	9	1
9	9	9	1
9	9	9	9
9	9	9	9
9	9	9	9
9	9	9	2
9	9	9	1
9	9	9	2

Nonsurviving Patients

Table 5.4 (cont.)

Cluster 3 (56 elements)

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	2	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	2	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	2	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

Nonsurviving Patients

Table 5.5: GPFN Classification.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.0000	1.0000	2.0000	0.0003	99.9700	-1.0443	4.4280
2.0000	1.0000	1.0000	0.9996	0.0379	-0.0491	95.0930
3.0000	1.0000	1.0000	1.0000	0.0004	-0.0799	92.0101
4.0000	1.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
5.0000	1.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
6.0000	1.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
7.0000	1.0000	1.0000	0.9999	0.0096	-0.0000	99.9998
8.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
9.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
10.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
11.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
12.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
13.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
14.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
15.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
16.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
17.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
18.0000	1.0000	2.0000	0.0002	99.9800	-0.7643	23.5658
19.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
20.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
21.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
22.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
23.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
24.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
25.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159

Column (1): Patient No. (In clustering order).
Column (2): Correct Class.
Column (3): Assigned Class.
Column (4): Response to Surviving Class assignment.
Column (5): Percent error corresponding to Surviving Class assignment.
Column (6): Response to Nonsurviving Class assignment.
Column (7): Percent error corresponding to Nonsurviving Class assignment.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
26.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
27.0000	1.0000	2.0000	0.0002	99.9805	-0.5738	42.6219
28.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
29.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
30.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
31.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
32.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
33.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
34.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
35.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
36.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
37.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
38.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
39.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
40.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
41.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
42.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
43.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
44.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
45.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
46.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
47.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
48.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
49.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
50.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159

(1)	(2)	(3)	(4)	(5)	(6)	(7)
51.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
52.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
53.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
54.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
55.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
56.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
57.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
58.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
59.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
60.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
61.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
62.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
63.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
64.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
65.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
66.0000	1.0000	2.0000	0.0002	99.9800	-0.7643	23.5658
67.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
68.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
69.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
70.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
71.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
72.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
73.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
74.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
75.0000	1.0000	1.0000	1.0032	0.3239	-1.0062	0.6159

(1)	(2)	(3)	(4)	(5)	(6)	(7)
76.0000	1.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
77.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
78.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
79.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
80.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
81.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
82.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
83.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
84.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
85.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
86.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
87.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
88.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
89.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
90.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
91.0000	1.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
92.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
93.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
94.0000	1.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
95.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
96.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
97.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
98.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
99.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
100.0000	1.0000	1.0000	1.0000	0.0004	-1.0134	1.3402

(1)	(2)	(3)	(4)	(5)	(6)	(7)
101.0000	2.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
102.0000	2.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
103.0000	2.0000	2.0000	0.0000	100.0000	-0.0000	100.0000
104.0000	2.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
105.0000	2.0000	2.0000	0.0000	100.0000	-0.6152	38.4772
106.0000	2.0000	2.0000	0.0000	100.0000	-0.9952	0.4836
107.0000	2.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
108.0000	2.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
109.0000	2.0000	2.0000	0.0000	100.0000	-0.9674	3.2570
110.0000	2.0000	2.0000	0.0003	99.9700	-1.0443	4.4280
111.0000	2.0000	2.0000	0.0000	100.0000	-0.0000	100.0000
112.0000	2.0000	2.0000	0.1183	88.1680	-0.9758	2.4250
113.0000	2.0000	2.0000	0.0000	100.0000	-0.0000	100.0000
114.0000	2.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
115.0000	2.0000	1.0000	1.0032	0.3239	-1.0062	0.6159
116.0000	2.0000	2.0000	0.0000	100.0000	-0.6305	36.9480
117.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
118.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
119.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
120.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
121.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
122.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
123.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
124.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
125.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687

(1)	(2)	(3)	(4)	(5)	(6)	(7)
126.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
127.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
128.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
129.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
130.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
132.0000	2.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
133.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
134.0000	2.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
135.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
136.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
137.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
138.0000	2.0000	1.0000	1.0000	0.0004	-1.0134	1.3402
139.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
140.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
141.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
142.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
143.0000	2.0000	2.0000	0.0001	99.9889	-0.9963	0.3687
144.0000	2.0000	2.0000	1.0003	0.0326	-0.9998	0.0176
145.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
146.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
147.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
148.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
149.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
150.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000

Table 5.5 (cont.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
151.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
152.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
153.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
154.0000	2.0000	1.0000	0.9999	0.0096	-0.0000	99.9998
155.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
156.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
157.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
158.0000	2.0000	1.0000	0.5945	40.5501	-0.0000	99.9999
159.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
160.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
161.0000	2.0000	1.0000	0.5945	40.5501	-0.0000	99.9999
162.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
163.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
164.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
165.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
166.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
167.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
168.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
169.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
170.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
171.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
172.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
173.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
174.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
175.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000

(1)	(2)	(3)	(4)	(5)	(6)	(7)
176.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
177.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
178.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
179.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
180.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
181.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
182.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
183.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
184.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
185.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
186.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
187.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
188.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
189.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
190.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
191.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
192.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
193.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
194.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
195.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
196.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
197.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
198.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
199.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000
200.0000	2.0000	2.0000	1.0000	0.0001	-1.0000	0.0000

Table 5.6: Direct Encoding Classification.

(1)	(2)	(3)	(4)	(5)
1.0000	1.0000	1.0000	67.0427	7.2231
2.0000	1.0000	1.0000	16.1420	3.6717
3.0000	1.0000	1.0000	67.0427	7.2231
4.0000	1.0000	1.0000	1.6065	0
5.0000	1.0000	1.0000	67.0427	7.2231
6.0000	1.0000	1.0000	67.0427	7.2231
7.0000	1.0000	1.0000	67.0427	7.2231
8.0000	1.0000	1.0000	1.6065	0
9.0000	1.0000	1.0000	67.0427	7.2231
10.0000	1.0000	1.0000	67.0427	7.2231
11.0000	1.0000	1.0000	67.0427	7.2231
12.0000	1.0000	1.0000	67.0427	7.2231
13.0000	1.0000	1.0000	67.0427	7.2231
14.0000	1.0000	1.0000	41.9277	4.3810
15.0000	1.0000	1.0000	67.0427	7.2231
16.0000	1.0000	1.0000	67.0427	7.2231
17.0000	1.0000	1.0000	67.0427	7.2231
18.0000	1.0000	1.0000	23.2131	9.6718
19.0000	1.0000	1.0000	23.2131	9.6718
20.0000	1.0000	1.0000	23.2131	9.6718
21.0000	1.0000	1.0000	67.0427	7.2231
22.0000	1.0000	1.0000	23.2131	9.6718
23.0000	1.0000	2.0000	3.6065	54.8196
24.0000	1.0000	1.0000	67.0427	7.2231
25.0000	1.0000	1.0000	23.2131	9.6718
26.0000	1.0000	1.0000	67.0427	7.2231
27.0000	1.0000	1.0000	67.0427	7.2231
28.0000	1.0000	1.0000	67.0427	7.2231
29.0000	1.0000	1.0000	23.2131	9.6718
30.0000	1.0000	1.0000	23.2131	9.6718
31.0000	1.0000	1.0000	23.2131	9.6718
32.0000	1.0000	1.0000	41.5281	4.6136
33.0000	1.0000	1.0000	67.0427	7.2231
34.0000	1.0000	1.0000	67.0427	7.2231
35.0000	1.0000	1.0000	67.0427	7.2231
36.0000	1.0000	1.0000	67.0427	7.2231
37.0000	1.0000	1.0000	67.0427	7.2231
38.0000	1.0000	1.0000	67.0427	7.2231
39.0000	1.0000	1.0000	67.0427	7.2231
40.0000	1.0000	1.0000	23.2131	9.6718
41.0000	1.0000	1.0000	23.2131	9.6718
42.0000	1.0000	2.0000	1.0000	14.0000
43.0000	1.0000	1.0000	67.0427	7.2231
44.0000	1.0000	1.0000	67.0427	7.2231
45.0000	1.0000	1.0000	23.2131	9.6718
46.0000	1.0000	1.0000	67.0427	7.2231
47.0000	1.0000	1.0000	67.0427	7.2231
48.0000	1.0000	1.0000	23.2131	9.6718
49.0000	1.0000	1.0000	67.0427	7.2231
50.0000	1.0000	1.0000	67.0427	7.2231

Column (1): Patient No.

Column (2): Correct Class.

Column (3): Assigned Class.

Column (4): Response to Surviving Class assignment.

Column (5): Response to Nonsurviving Class assignment.

Table 5.6 (cont.)

(1)	(2)	(3)	(4)	(5)
51.0000	1.0000	1.0000	67.0427	7.2231
52.0000	1.0000	1.0000	67.0427	7.2231
53.0000	1.0000	1.0000	67.0427	7.2231
54.0000	1.0000	1.0000	23.2131	9.6718
55.0000	1.0000	1.0000	67.0427	7.2231
56.0000	1.0000	1.0000	67.0427	7.2231
57.0000	1.0000	1.0000	67.0427	7.2231
58.0000	1.0000	1.0000	67.0427	7.2231
59.0000	1.0000	1.0000	67.0427	7.2231
60.0000	1.0000	1.0000	67.0427	7.2231
61.0000	1.0000	1.0000	23.2131	9.6718
62.0000	1.0000	2.0000	3.6065	54.8196
63.0000	1.0000	1.0000	67.0427	7.2231
64.0000	1.0000	1.0000	15.3437	12.8196
65.0000	1.0000	1.0000	23.2131	9.6718
66.0000	1.0000	1.0000	67.0427	7.2231
67.0000	1.0000	1.0000	67.0427	7.2231
68.0000	1.0000	1.0000	67.0427	7.2231
69.0000	1.0000	1.0000	23.2131	9.6718
70.0000	1.0000	1.0000	15.3437	12.8196
71.0000	1.0000	1.0000	23.2131	9.6718
72.0000	1.0000	1.0000	23.2131	9.6718
73.0000	1.0000	1.0000	67.0427	7.2231
74.0000	1.0000	1.0000	67.0427	7.2231
75.0000	1.0000	1.0000	67.0427	7.2231
76.0000	1.0000	1.0000	67.0427	7.2231
77.0000	1.0000	1.0000	67.0427	7.2231
78.0000	1.0000	2.0000	3.6065	54.8196
79.0000	1.0000	1.0000	67.0427	7.2231
80.0000	1.0000	1.0000	67.0427	7.2231
81.0000	1.0000	1.0000	67.0427	7.2231
82.0000	1.0000	1.0000	67.0427	7.2231
83.0000	1.0000	1.0000	23.2131	9.6718
84.0000	1.0000	1.0000	67.0427	7.2231
85.0000	1.0000	1.0000	23.2131	9.6718
86.0000	1.0000	1.0000	67.0427	7.2231
87.0000	1.0000	1.0000	67.0427	7.2231
88.0000	1.0000	1.0000	41.9277	4.3810
89.0000	1.0000	1.0000	67.0427	7.2231
90.0000	1.0000	1.0000	23.2131	9.6718
91.0000	1.0000	1.0000	67.0427	7.2231
92.0000	1.0000	1.0000	67.0427	7.2231
93.0000	1.0000	1.0000	67.0427	7.2231
94.0000	1.0000	1.0000	23.2131	9.6718
95.0000	1.0000	1.0000	67.0427	7.2231
96.0000	1.0000	1.0000	67.0427	7.2231
97.0000	1.0000	1.0000	67.0427	7.2231
98.0000	1.0000	1.0000	67.0427	7.2231
99.0000	1.0000	1.0000	67.0427	7.2231
100.0000	1.0000	2.0000	2.8196	33.8819

(1)	(2)	(3)	(4)	(5)
101.0000	2.0000	2.0000	3.6065	54.8196
102.0000	2.0000	1.0000	67.0427	7.2231
103.0000	2.0000	2.0000	3.6065	54.8196
104.0000	2.0000	2.0000	1.0000	14.0000
105.0000	2.0000	2.0000	3.6065	54.8196
106.0000	2.0000	2.0000	3.6065	54.8196
107.0000	2.0000	2.0000	3.6065	54.8196
108.0000	2.0000	1.0000	15.3437	12.8196
109.0000	2.0000	2.0000	3.6065	54.8196
110.0000	2.0000	2.0000	3.6065	54.8196
111.0000	2.0000	1.0000	15.3437	12.8196
112.0000	2.0000	1.0000	15.3437	12.8196
113.0000	2.0000	2.0000	3.6065	54.8196
114.0000	2.0000	2.0000	3.6065	54.8196
115.0000	2.0000	2.0000	2.8196	33.8819
116.0000	2.0000	2.0000	3.6065	54.8196
117.0000	2.0000	1.0000	15.3437	12.8196
118.0000	2.0000	1.0000	15.3437	12.8196
119.0000	2.0000	1.0000	15.3437	12.8196
120.0000	2.0000	2.0000	1.0000	14.0000
121.0000	2.0000	1.0000	15.3437	12.8196
122.0000	2.0000	2.0000	1.0000	14.0000
123.0000	2.0000	1.0000	15.3437	12.8196
124.0000	2.0000	1.0000	15.3437	12.8196
125.0000	2.0000	2.0000	1.0000	14.0000
126.0000	2.0000	2.0000	1.0000	14.0000
127.0000	2.0000	2.0000	3.6065	54.8196
128.0000	2.0000	2.0000	3.6065	54.8196
129.0000	2.0000	2.0000	2.1875	34.5140
130.0000	2.0000	1.0000	67.0427	7.2231
131.0000	2.0000	2.0000	0	2.0000
132.0000	2.0000	2.0000	1.0000	14.0000
133.0000	2.0000	1.0000	23.2131	9.6718
134.0000	2.0000	2.0000	3.6065	54.8196
135.0000	2.0000	1.0000	15.3437	12.8196
136.0000	2.0000	1.0000	23.2131	9.6718
137.0000	2.0000	2.0000	3.6065	54.8196
138.0000	2.0000	2.0000	2.1875	34.5140
139.0000	2.0000	2.0000	1.0000	14.0000
140.0000	2.0000	2.0000	3.6065	54.8196
141.0000	2.0000	2.0000	3.6065	54.8196
142.0000	2.0000	2.0000	1.0000	14.0000
143.0000	2.0000	2.0000	1.0000	14.0000
144.0000	2.0000	2.0000	3.6065	54.8196
145.0000	2.0000	2.0000	3.6065	54.8196
146.0000	2.0000	2.0000	3.6065	54.8196
147.0000	2.0000	2.0000	3.6065	54.8196
148.0000	2.0000	2.0000	3.6065	54.8196
149.0000	2.0000	2.0000	3.6065	54.8196
150.0000	2.0000	2.0000	3.6065	54.8196

Table 5.6 (cont.)

(1)	(2)	(3)	(4)	(5)
151.0000	2.0000	2.0000	3.6065	54.8196
152.0000	2.0000	2.0000	3.6065	54.8196
153.0000	2.0000	2.0000	3.6065	54.8196
154.0000	2.0000	2.0000	3.6065	54.8196
155.0000	2.0000	2.0000	3.6065	54.8196
156.0000	2.0000	2.0000	3.6065	54.8196
157.0000	2.0000	2.0000	3.6065	54.8196
158.0000	2.0000	2.0000	3.6065	54.8196
159.0000	2.0000	2.0000	3.6065	54.8196
160.0000	2.0000	2.0000	3.6065	54.8196
161.0000	2.0000	2.0000	3.6065	54.8196
162.0000	2.0000	1.0000	67.0427	7.2231
163.0000	2.0000	2.0000	0	1.0000
164.0000	2.0000	2.0000	3.6065	54.8196
165.0000	2.0000	2.0000	0.6065	2.2131
166.0000	2.0000	1.0000	67.0427	7.2231
167.0000	2.0000	1.0000	67.0427	7.2231
168.0000	2.0000	2.0000	3.6065	54.8196
169.0000	2.0000	2.0000	0.3679	1.9744
170.0000	2.0000	2.0000	3.6065	54.8196
171.0000	2.0000	1.0000	23.2131	9.6718
172.0000	2.0000	2.0000	3.6065	54.8196
173.0000	2.0000	1.0000	16.1420	3.6717
174.0000	2.0000	2.0000	0	1.0000
175.0000	2.0000	2.0000	3.6065	54.8196
176.0000	2.0000	2.0000	3.6065	54.8196
177.0000	2.0000	2.0000	3.6065	54.8196
178.0000	2.0000	2.0000	3.6065	54.8196
179.0000	2.0000	2.0000	3.6065	54.8196
180.0000	2.0000	2.0000	3.6065	54.8196
181.0000	2.0000	2.0000	3.6065	54.8196
182.0000	2.0000	2.0000	0.1353	1.1353
183.0000	2.0000	2.0000	1.0000	14.0000
184.0000	2.0000	2.0000	3.6065	54.8196
185.0000	2.0000	2.0000	1.0000	14.0000
186.0000	2.0000	2.0000	3.6065	54.8196
187.0000	2.0000	2.0000	3.6065	54.8196
188.0000	2.0000	2.0000	1.0000	14.0000
189.0000	2.0000	2.0000	3.6065	54.8196
190.0000	2.0000	2.0000	0	2.0000
191.0000	2.0000	2.0000	1.0000	14.0000
192.0000	2.0000	2.0000	3.6065	54.8196
193.0000	2.0000	2.0000	3.6065	54.8196
194.0000	2.0000	1.0000	15.3437	12.8196
195.0000	2.0000	2.0000	3.6065	54.8196
196.0000	2.0000	2.0000	3.6065	54.8196
197.0000	2.0000	1.0000	67.0427	7.2231
198.0000	2.0000	2.0000	1.0000	14.0000
199.0000	2.0000	1.0000	67.0427	7.2231
200.0000	2.0000	2.0000	0	1.0000

Chapter 6

Conclusions

This Phase I project demonstrated the use of a Gaussian Potential Function Network for classification of trauma care data. The summary of the project effort and accomplishments as well as recommendations for future work are as follows.

6.1 Summary of Research Effort and Accomplishments

- A Gaussian Potential Function Network (GPFN) architecture was created that allows the differentiation of patient categories that correspond to trauma severity levels. These classes constitute the basis for field triage. The GPFN is based on a collection of Guassian Potential Function Units (GPFUs) that are positioned at feature space locations characterized by the statistics of the data distributions such as the mean and the standard deviation.

- The GPFN has been shown to be "trainable" through modification of the amplitudes, the means and the covariance matrices of the GPFUs so as to allow a desired class integer declaration.

- The fuzzy c-means clustering algorithm was employed to separate the data into possible different groups representing their natural spatial distribution. An additional algorithm was used to establish the most likely number of clusters. This information is used with the fuzzy c-means algorithm which requires a priori specification of the number of clusters anticipated in the data.

- An additional classification approach was presented that is simple and direct. It involves assignment of a gaussian function to each data point of a given class. This method allows the direct representation of the frequency of occurrence of feature vectors among the various classes and has a foundation in the multidimensional probability density estimation techniques.

The results obtained provide a solid basis and powerful tools for trauma care classification efforts. This Phase I research also opens several opportunities for further investigations of the challenging problems faced by the important field of trauma care classification.

6.2 Recommendations for Future Work

6.2.1 Expanded Data Base

To capture the core statistical validity of the trauma care classification problem there is a need to expand its dependency on an extensive data base. The data base must contain the largest possible number of past records compatible with the Army's expected utilization scenarios so that the classification answers have a firm foundation in past observations. The classification algorithms examined in the Phase I effort provide a faithful depiction of the statistical prevalence of past feature vectors. This is in contrast with other approaches which impose mathematical constructs that may not always be truly representative of past experience as reflected in the data structure.

6.2.2 Feature Set Selection

The trauma care classification act is effected through a set of variables that have been found by medical researchers through past experience and knowledge as being useful for such an act. Variables are related to vital signs (such as, pulse, blood pressure and level of consciousness) as key determinants of organ and tissue damage. Variables are used that are linked to cardiovascular, respiratory and central nervous system functions. Variables that have been investigated as correlating with trauma care classification purposes include pulse, skin color, bleeding, injury region, injury type, respiratory rate, respiratory expansion, systolic blood pressure, capillary refill, eye opening, best verbal response and best motor response. It is of interest to investigate which variables, in combination, among the many proposed in the past, provide the best predictive capabilities for trauma care classification. This can be done within the classification framework of the Phase I results.

6.2.3 Trauma Care Classification Scores

Various trauma scores have been created through the years in an attempt to capture by means of field measurable variables the degree of trauma severity. Among the most prominent efforts in this area are Dr. H. Champion's Trauma Score (TS), the Abbreviated Injury Scale (A.I.S.) published in 1971 as a single comprehensive system for rating tissue damage sustained in motor-vehicle accidents, the Injury Severity Score (ISS) developed in 1974 to evaluate motor-vehicle victims with multiple injuries, the CRAMS scale and others. These scores attempt to categorize the degree of severity of trauma patients and some (such as the TS and CRAMS) are specifically designed for field triage of trauma victims to trauma centers. It is of significant interest to correlate the classification declarations of the algorithms developed in this effort with the corresponding major trauma score values. This will provide a substantive validation and enhance acceptance of both the classification approach and the successful trauma related score formulations.

6.2.4 Hardware for Field Use

A computer-like small, portable, trauma care classification system will find an important use by the Army in the field. The constantly improving state-of-the-art techniques in hardware design, miniaturization and manufacture of circuits, circuit boards, signal processing and software programming make such a system easy-to-use and convenient-to-carry. The system will acquire, process, display, record and store trauma care related information. Also, the system will provide extended output ports to peripheral devices, such as PC computers, pen recorders, and display monitors for post data processing and analysis, ink recording and large screen displaying. The objective is to produce a general and useful device to allow trauma care classification to be effected in the field environment. The software will not only be capable of accommodating the classification algorithms developed under this project but will also be able to compute and display any desired trauma score, such as TS or ISS, given the corresponding input variables values.

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